MOVES Conference 2013
Research Talks

Stephen Abbott, Middlebury College: ***Take a walk on the Boardwalk***

What is the best Monopoly? Scientific American columnist Ian Stewart attempted a partial answer to this question, determining that all spaces are visited with the same frequency – a conclusion any experienced Monopoly player knows is false. Using Markov chains and a little ingenuity, it is possible to vastly improve on Stewart's analysis.

Max Alekseyev, University of South Carolina: ***Solving the Tower of Hanoi with random moves***

What is the expected number of moves required to solve the Tower of Hanoi puzzle if every move is chosen uniformly random from the set of all possible moves? I will present solutions to this and related puzzles and explore their connection to random walks in fractal graphs.

Elizabeth Arnold, James Madison University: k-*Potent Groebner basis computations for Sudoku*

Sudoku can be described as a system of polynomials which can then be solved using Groebner basis techniques. A Boolean idempotent approach restricts degree growth of intermediate polynomials, but increases the number of variables. We propose a k-potent approach which also attains this goal, while minimizing the number of variables.

Jennifer Beineke, Western New England University and Lowell Beineke, Indiana University - Purdue University Fort Wayne: ***Some of the A-B-Cs (and Ds) of graphs and games***

Exploring the mathematics behind a game can be as much fun as playing it. Graph theory has proved to be useful for analyzing many games and puzzles, whether for determining winning strategies, or simply for understanding the outcomes. In this article, we explore some ABC's of finding graphs in games.

Leigh Marie Braswell, Phillips Exeter Academy: ***The Cookie Monster Problem***

A cookie monster is given a set of cookie jars. In one move he can choose any subset of jars and take the same number of cookies from those jars. We analyze the minimum number of moves needed to empty all the jars for certain distributions of cookies.

Tricia Brown, Armstrong Atlantic State University: ***Non-attacking arrangements of n queens with initial placements***

To begin to answer a question posed by Bell and Stevens, we give constructions of general solutions to the n queens problem in the case n congruent to zero modulo 6 and in the case n congruent to two modulo 6.

Maureen Carroll, University of Scranton: ***Tic-tac-toe on an affine plane***

After introducing finite affine planes, we extend the standard game of tic-tac-toe to play on a plane of order n. We discuss the orders for which the first player has a winning strategy and those where the second player has a drawing strategy.

Doug Chatham, Morehead State University: ***The* N-k *Queens Problem***

The class N Queens Problem calls for placements of N mutually non-attacking queens on an N-by-N chessboard. How many squares do we need to forbid queen placement from in order to reduce the maximum number of mutually non-attacking queens by some given k?

Erik Demaine, Massachusetts Institute of Technology: ***Geometric puzzles: algorithms and complexity***

I love geometry because the problems and solutions are fun and often tangible. Puzzles are one way to express these two features, and are also a great source of their own computational geometry problems: which puzzles can be solved and/or designed efficiently using computer algorithms? Proving puzzles to be computationally difficult leads to a mathematical sort of puzzle, designing gadgets to build computers out of puzzles. I will describe a variety of algorithmic and computational complexity results on geometric puzzles, focusing on more playful and recent results.

Donna Dietz, American University: ***Spot it! Solitaire***

The card game of Spot it! (by Blue Orange Games) contains 55 circular cards with various symbols on them. Players attempt to find matching symbols between cards. In this talk, a different kind of challenge is presented, for a single player! Try to arrange 49 of the 55 cards in a 7 by 7 grid so that each row, each column, and each diagonal (of both types) has a common symbol in it!

Suzanne Dorée, Augsburg College: *The graphs of Hanoi: visualizing solutions to the Tower of Hanoi puzzle*
The classic Tower of Hanoi puzzle has regained popularity as an "App". The lesser-known, beautifully symmetric Graphs of Hanoi make it easy to see a solution and provide a powerful tool for research, especially multi-peg generalizations. This talk provides an introduction to the power of (combinatorial) graphs as mathematical models.

Noam Elkies, Harvard University: *The Numberplay arithmetic progression challenge*
It has been known for some time that any two-coloring of the whole numbers from 1 through 9 must contain at least one monochromatic three-term arithmetic progression (MAP3). Ron Graham asked some decades ago, and recently again via the Numberplay blog of the New York Times, whether there is any set of only eight whole numbers with the same property (the set {1,2,3,4,5,6,7,8} does not, as witness the coloring BBRRBBRR; but other examples are harder: try {0,2,4,5,6,7,8,10}, which has a unique MAP3-free two-coloring). He conjectured that there are none, that is, that any set of 8 numbers has at least one MAP3-free two-coloring. We describe how we proved this by reducing the problem to a computation that was not just finite but tractable: only 2097152 cases (where at first it seemed to be more by 10+ orders of magnitude). The observation that 2097152 is an exact power of 2 led in turn to a series of further formulas in enumerative combinatircs, at first conjectural and eventually proved by Fu Liu and Richard Stanley. This story illustrates the unexpected connections and collaboration often seen in modern mathematical research.

Robert Fathauer, Tessellations: *Elliptical and hyperbolic fractal tilings*
Fractal tilings are explored in which the sum of the angles meeting at a vertex is greater than (elliptical case) or less than (hyperbolic case) 360 degrees. Examples have been constructed in Euclidian 3-space using paper models and/or Mathematica. These exhibit complex and unique surfaces.

Darren Glass, Gettysburg College: *Chip firing puzzles on graphs*
In this talk, we introduce a family of puzzles that can be played on graphs. While these puzzles can be created and played using nothing more than a pencil, paper addition, and subtraction, there is deep underlying mathematics that we will see glimpses of. This work leads to interesting questions that combine linear algebra, group theory, graph theory, and algebraic geometry.

Susan Goldstine, St. Mary's College of Maryland: *Tessellations on bead crochet bracelets*
Bead crochet bracelets are easy to wear, fun to make, and astonishingly hard to design, without math. Learn how to use simple but clever geometry to turn a tessellation of a flat plane of beads into a stunning, seamless, edgeless bracelet!

Robert Grober, Yale University, Stephen Della Pietra, Renaissance Technologies, and Vincent Della Pietra, Renaissance Technologies: *Universal trajectory of a ball rolling on a tilted, planar surface*
A ball rolling on a tilted, planar surface subject to a constant drag force approximates a golf ball rolling across a putting green near to the hole. An analytic solution to these non-linear equations of motion is derived, revealing that all putt trajectories are derived from a universal trajectory.

John Harris, Furman University: *Bobo's favorite card trick*
J.B. Bobo was one of the most prolific magicians of the 20th century. Although he is known primarily for his classic book on magic with coins, it is his favorite card trick that is the topic of this presentation. Mathematics is what makes this impromptu, self-working trick seem so impossible.

Brian Hopkins, Saint Peter's University: *Henryk Eriksson's variation on Bulgarian solitaire*
The Bulgarian Solitaire game was named by the Swedish mathematician Henryk Eriksson, who also brought it to the US, where Martin Gardner popularized it in one of his last Scientific American articles. Using the language of integer partitions, we analyze a variation Eriksson introduced.

Jeff Johannes, SUNY Geneseo: *Game: SET, and math*
The card game SET is simple to learn, and often children excel at this game. Hiding within this game is a vast amount of mathematics. In this talk we will find not only the expected probability and combinatorics, but also some geometry, algebra, and topology.

Brant Jones, James Madison University: *Solitaire Mancala games and the Chinese Remainder Theorem*
Mancala is a generic name for a popular family of sowing games. Many two-player versions allow a player to move again if their move ends in the store. We study a simple solitaire mancala game that facilitates the analysis of these "sweep" moves, where all of the stones are captured.

Tanya Khovanova, Massachusetts Institute of Technology: ***Modern coin weighing puzzles***

There are many coin weighing puzzles. I will concentrate on two puzzles inspired by cryptography and parallel computing.

Klay Kruczek, Southern Connecticut State University: ***Using fractional matchings to pairing strategy draws in N^d Tic-tac-toe***

It is known Player 2 cannot win any Tic-tac-toe games. Given a fixed dimension d, we will discuss for what values of N a Pairing Strategy Draw (PSD) in N^d Tic-tac-toe exists for Player 2. The best possible result is $N \geq (2/(\ln 2))(d) - 1$. We will show a PSD exists when $N \geq 3d - O(sqrt(d))$.

Dominic Lanphier, Western Kentucky University: ***Duels, truels, gruels, and survival of the unfittest***

Duels are types of two person games where each player shoots under various conditions. A truel is a three person duel. We look at various generalizations of these games, such as where each player can be hit a number of times. We also look at multi-player duels.

Anany Levitin, Villanova University: ***One-move puzzles with mathematical content***

The talk's topic is puzzles with some mathematical content that can be solved in one move. Both well-known and lesser-known such puzzles will be discussed. A few of them exploit nontrivial mathematical facts and are what Peter Winkler has called "puzzles you think you must not have heard correctly."

Eliana Lorch, Museum of Mathematics: ***The Kaprekar Routine: an exploration of patterns***

At first, the Kaprekar Routine seems like a party trick, always returning 6174. This paper uses the techniques for small numbers to extend the Routine to numbers of greater lengths and in different bases. Additionally, this paper focuses on finding the bigger picture through symmetries and patterns, concluding with a unique proof on divisibility.

John Lorch, Ball State University: ***Super-orthogonal Sudoku***

In this talk we introduce the notion of super-orthogonality for sudoku, and see that the size of complete sets of super-orthogonal sudoku solutions is tied to the existence of a certain class of ordered orthogonal arrays.

Stephen Lucas, James Madison University: ***Representing numbers using Fibonacci variants***

Every natural number has a unique representation as a sum of Fibonacci numbers, with the constraint that no consecutive pair is chosen. We show where this representation is useful, extend it to representing arbitrary reals via a continued fraction representation, and make it more efficient using generalizations of Fibonacci numbers.

Elizabeth McMahon, Lafayette College: ***Error detection in the card game SET***

We begin with a variation of the game of SET, which makes use of a parity check. This allows us to detect (but not correct) one error made in the play of the game. Further analysis leads us to define a Hamming distance on the deck of cards; we can use this distance to classify different categories of non-sets, three cards that do not form a set.

John McSweeney, Rose-Hulman Institute of Technology: ***Graph theory-based analysis of crossword puzzle difficulty***

We describe a way to quantify the degree of difficulty of a crossword puzzle by considering the clue difficulty and the grid structure, by modeling these using a random distribution and a network (graph) structure, respectively. Simulations on actual puzzles from the New York Times will be presented.

David Molnar, Felician College: ***Connection games and Sperner's Lemma***

Atoll is a connection game, played on a grid of hexagons surrounded by eight 'islands'. The goal is to connect two opposite islands of one's color. It can be proven that someone must win at *Hex* using Sperner's Lemma. I generalize this result to *Atoll* and infinitely many other games.

Colm Mulcahy, Spelman College: ***What's the deal? Mathematics inspired by dealing cards into a pile***

The first "Card Colm" at MAA.org in 2004, dedicated to Martin Gardner on his 90th birthday, was about a surprising new card dealing principle, which lead to fun mathemagical effects. We'll discuss some recent generalizations, and entertaining spelling effects they suggest.

David Nacin, William Paterson University: ***On a complex valued Sudoku***

In the October 2010 entry of the blog Bit-player, a four-by-four KenKen over a set of complex numbers was posted with mention that the uniqueness of the solution had not been checked. We show uniqueness of their solution and classify all KenKen with similar conditions having unique solutions.

Jonathan Needleman, Le Moyne College: ***Boggle logic puzzles: new solutions, and even more questions***
Boggle logic puzzles are a kind of reverse word search. I will summarize work on both minimal and maximal solutions to these puzzles. Afterward, I will describe a few open problems, and relate these problems to questions in graph theory.

Kenneth Price, University of Wisconsin Oshkosh: ***Arrowgrams over finite groups***
Arrowgrams are puzzles you solve by labeling arrows with group elements using transitivity. The arrows connect letters, which you use to decode a secret message. This talk will give an account of arrowgram constructions that depend on the choice of the underlying finite group.

David Richeson, Dickinson College: ***Making the impossible possible: how to trisect an angle***
In 1837 Wantzel famously proved that it is impossible to trisect an arbitrary angle using a compass and straightedge. In this talk we present a collection of ingenious methods to trisect angles using implements such as a marked straightedge, origami, specially-shaped tools, exotic curves, and the hands of a clock.

Jason Rosenhouse, James Madison University: ***Non-classical knights and knaves***
Puzzles about liars and truthtellers have a long history. They have tremendous pedagogical value for introducing people to sophisticated ideas in logic. We shall introduce these ideas with some entertaining examples, and will then discuss possible extensions of the classical puzzles to non-classical logics.

Paul Salomon, John Burroughs School: ***Problems of imbalance***
This paper presents a twist on a well-known puzzle type: balanced scale problems. We consider a series of problems in which the scales do not balance, and hence access the logical reasoning required for systems of linear inequalities. We also present the wonderful results of an imbalance problem-writing contest.

Henry Segerman, Oklahoma State University: ***Puzzling the 120-cell***
After reviewing the construction of the 120-cell, and its combinatorial Hopf fibration, we give a collection of six ribs: parts of the stereographic projection of the rings of the fibration. We use these ribs to build a variety of burr puzzles.

Derek Smith, Lafayette College: ***Solving generalizations of the Slothouber-Graatsma puzzle***
The Slothouber-Graatsma cube-packing puzzle, also attributed to Conway, can be generalized to cubes of odd side length larger than 3. For an infinite family of such puzzles, containing versions due to O'Beirne and Conway and Thiessen, we show that solutions are unique up to symmetry.

Ron Taylor, Berry College: ***Sequential mathematical games based on additive and subtractive color mixing arithmetic***
In this presentation we will introduce two sequential games whose rules are mathematical in nature, though no explicit mathematics is necessary during game play. Both games are based on color mixing rules which can be represented by a well known algebraic structure with interesting geometric and set theoretic interpretations.

Bruce Torrence, Randolph-Macon College: ***Lights Out for gamers and mathematicians***
The game Lights Out has inspired an impressive mathematical literature. In this presentation we will explore some fiendishly clever strategies for attacking Lights Out puzzles, some developed by gamers, others developed by mathematicians, and we will see how the two approaches have informed and influenced each other.

Robert Vallin, Slippery Rock University: ***From the Gilbreath Principle to new types of numbers***
The Gilbreath Principle was introduced in 1958 as a basis for card tricks. We discuss the magical origins with a trick, then show how it leads to particular permutations of 1, 2, N. We apply the permutations to create subsets of [0,1] and discuss what falls under Gilbreath Numbers.

Peter Winkler, Dartmouth College: ***Should you be happy?***
You've received mixed news: should you be happy? Yes, if you're better off than you were before, but sometimes it's not so easy to determine that. We illustrate the problem with a case where intuition argues both ways – and a trick to help determine which argument is better.

Carolyn Yackel, Mercer University: *Symmetry group of the tetraflexagon*

The actions on paper toy flexagons should naturally form groups. Surprisingly, this is not always true. When the actions do afford a group structure, the full symmetry groups are often much stranger than would initially appear. We discuss history, challenges, and results of this deceptive issue for the square flexagon.

Sam Vandervelde, St. Lawrence University: *The world's hardest elementary domino tiling problems*

Tiling problems have long delighted recreational mathematicians due to their potential for elementary formulation yet unexpected difficulty and elegance. We survey a sampling of classic and less well-known tiling puzzles, then examine more closely one such problem that appeared on the 2009 USAMO, successfully solved by a single student.

Jade Vinson, Renaissance Technologies: *Shapes of space: the challenge of negative curvature*

The 'Shapes of Space' exhibit at MoMath explores surfaces of constant Gaussian curvature. Negative curvature is the wildest and most exciting – but also the most challenging to demonstrate physically. We discuss the mathematical limits of what is possible, the final design and alternatives we considered, and open questions.

Mathematical Activities

Skona Brittain, SB Family School and SB Crafts: *The most MatheMagical number*

Several magic tricks involving the most MatheMagical number will first be presented, then figured out or revealed, and then mathematically analyzed. Participants will practice performing the tricks themselves and will make their own set of MatheMagics cards.

Ethan Brown, Phillips Academy Andover: *The dynamic world of mathematics*

When you look on the news for new discoveries, mathematics isn't something you expect to find. However, there are many new things to learn in math. The speaker will present some interesting, recently invented mathematical concepts and ideas, showing the importance and beauty of mathematics.

Michael Dorff, Brigham Young University: *Shortest paths, soap films, and mathematics*

What do shortest paths, soap films, and mathematics have in common? We will explore this topic with pictures, guesses, glass plates and rigged frames dipped in soap solution, computer animations, and objects constructed by a 3-d printer.

Patrick Honner, Brooklyn Technical High School: *Sphere dressing*

In this activity, participants will attempt to "dress" a sphere by creating a hat that satisfies various geometric requirements. A variety of challenges will be offered, and materials will be provided.

Jeff Johannes, SUNY Geneseo: *Kaleidoscopic mathematics*

We will play with mirrors to see what makes some kaleidoscopes look more appealing. We will experiment with several mirror arrangements, including many different arrangements from the most common kaleidoscopes. For each arrangement we will look at examples of actual kaleidoscopes with those arrangements, in addition to our own handmade models.

Tanya Khovanova, Massachusetts Institute of Technology: *SET game theory*

The activity is targeted for high-school students or anyone who likes the game of SET. It also can include people who want to learn about the game of SET.

Kevin Lee, Normandale Community College: *TesselManiac and The Flipping Tile Game*

The Flipping Tile Game is part of my new program TesselManiac that creates Escher-like tessellations. I will demonstrate how The Flipping Tile Game enables students to explore and understand in an animated and engaging way the four types of isometries: translation, rotation, reflection, and glide reflection, as well as their compositions.

Ron Lancaster, University of Toronto: *Math Trail*

A mathematical morning walk through Madison Square Park, starting at MoMath. Fun for all ages.

Ron Lancaster, University of Toronto: *Survivor: a card puzzle and a magic trick*

We will eliminate playing cards from a pile in a certain way. We will answer questions such as which card survives and how does that card depend on how many cards are in the pile. We will perform an amazing mathematical magic trick based on the card that survives.

David Molnar, Felician College: *Connection games*

Connection games are a family of 2-player perfect information games. Participants will have the opportunity to play a variety of connection games, ranging from the classic *Hex* and *The Game of Y*, to more recent discoveries such as *Atoll* and *Catchup*.

Saba Nafees and Udaya Jayatilake, Texas Tech University: *Making polyhedra: a hands-on experience*

Polyhedra are solid figures bounded by plane faces. Our focus will be on convex polyhedra whose faces are regular polygons. There are three types: Platonic, Archimedean, and Johnson. We will demonstrate how to construct these by drawing polygons, cutting outlines, and finally putting them together to reveal the 3-D structure.

Robin Schwartz, College of Mount Saint Vincent: *Move over Sudoku, your fun cousin Kakuro is sum puzzle!!*

If 3 boxes must add to 7, the numbers are 1, 2 and 4 as you cannot repeat a digit – the challenge is finding the correct order! This session will include the fun and confidence that comes from problem-solving.

Derek Smith, Lafayette College: *The Fitch-Cheney Five-Card Trick*

Learn and practice a purely mathematical card trick that will amaze your friends and family!

Ted Theodosopoulos, Saint Ann's School: *Math circle on chaotic dynamics*

In this talk I will share examples of games, incorporating strategic, stochastic and interactive components. These games are designed to help explore mathematical concepts, from coordinate geometry and place-value to martingales and quantum correlations.

Eve Torrence, Randolph-Macon College: *A workshop on stellation based sculpture*

In this workshop we will study the structure of some of the complex and beautiful sculptures by George Hart and others. We will learn about polyhedral stellation, which is fundamental to understanding how these sculptures are designed and constructed. Each participant will assemble a small sculpture kit designed by the presenter and based on this concept.

Ted Welsh, Westfield State University: *Graph theory on LEGO grids*

Two summers ago, I found a really neat way to build a grid of equilateral triangles using LEGO elements. I quickly realized there were piles of puzzling possibilities. Come try your hand at these graph theory puzzles, and see if you can build your own beautiful new solutions!

Brandy Wiegers, National Association of Math Circles: *Criss-Cross, exploring the Euler characteristic*

The game of Criss-Cross is played on a blank sheet of paper by two players. The game board is created by drawing four points at the vertices of a square, along with one to seven additional points anywhere in its interior. Players alternate turns drawing a single straight line segment joining any two points, as long as the segment does not pass through any other points or segments already appearing on the game board. The winner is the last player able to make a legal move. Where is the math in this you say? Come and find out!

Debbie Yuster, SUNY Maritime College: *Tower of Hanoi*

In this activity, we will play the Tower of Hanoi game, moving a stack of discs from one pole to another, according to the rules of the game. We'll discover a formula for the minimum number of moves needed, and will explore the mathematics behind the game.