MOMATH
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Proofs from THE BOOK: Putting the Pieces Together
Problems

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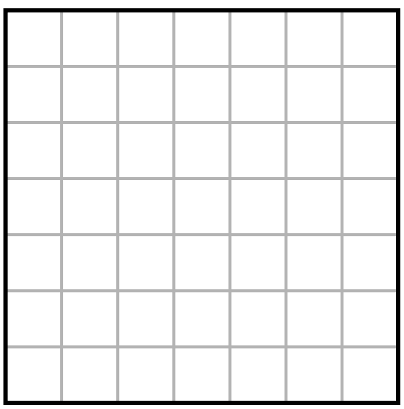
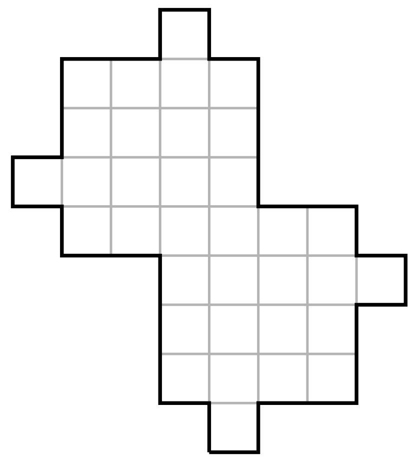
Problem 1: Cover the 7 x 7 board with dominos so that one square is left uncovered. Which squares are possible as “last uncovered square”?



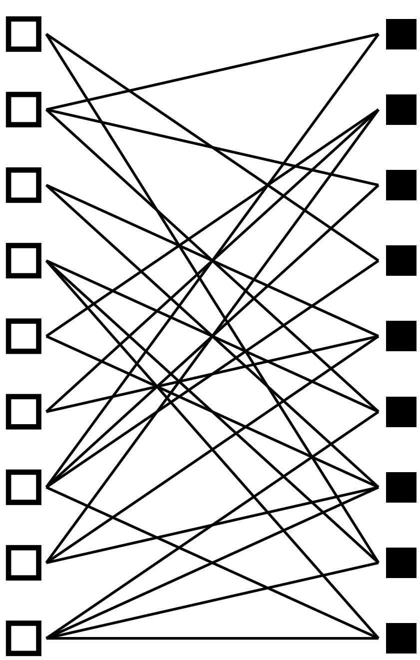
Problem 2: Here is a more interesting board. Can you cover it with dominos? If no, why not?
Can you offer a “book proof”?

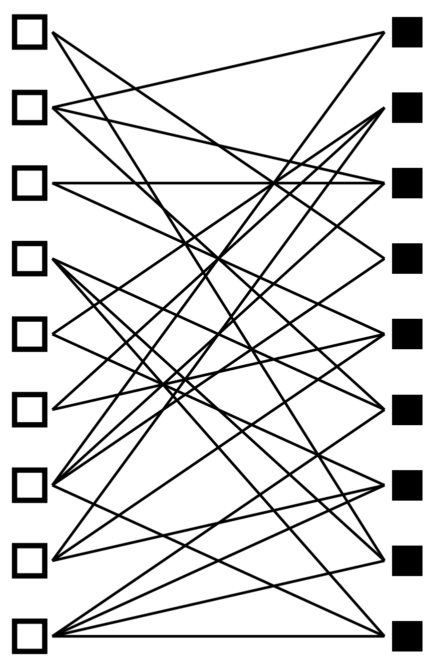


Problem 3: In which of the following is there a perfect “marriage arrangement” between the white squares and the black squares?

If there is one, is there a systematic method for finding one?

If there is none, can you give a short proof?





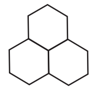
Problem 4: Which of the following boards T_6, T_7, T_8, and T_9 can you cover with triads T_2?

If it can be done, produce a solution!

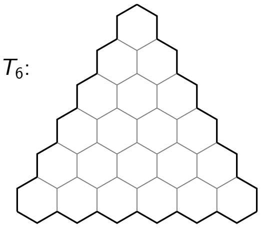
If it can't be done,

- is there a "book proof"?
- what is the smallest number of hexagons that will stay uncovered?
- is there a "± solution" where negative triads are allowed?

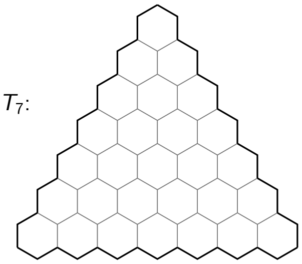
***T_2*:**



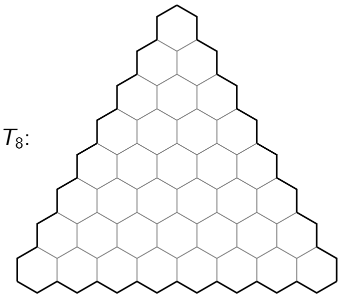
T_6:



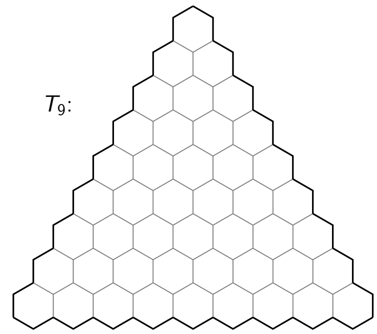
T_7:



T_8:

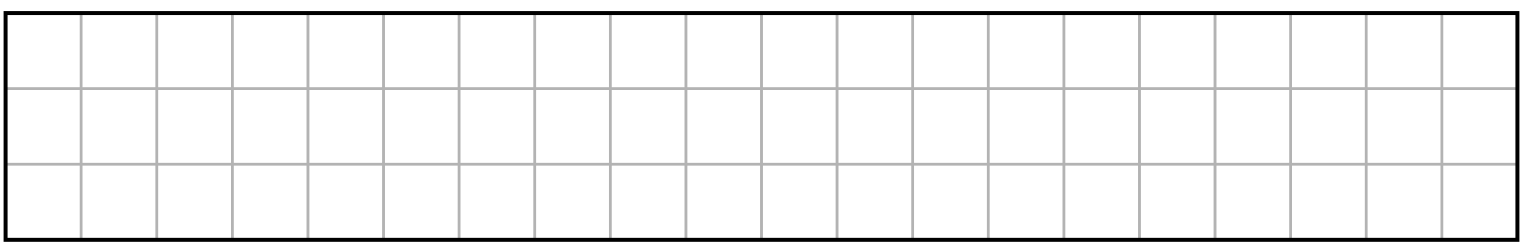


T_9:

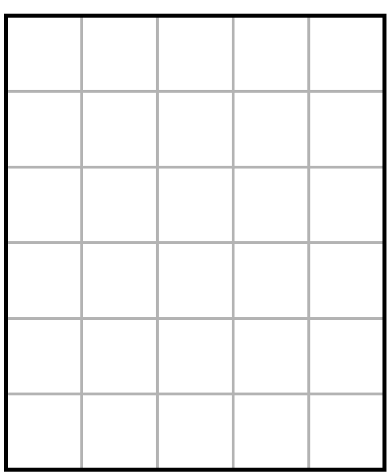


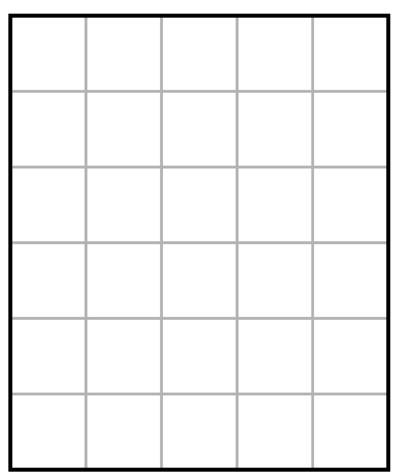
Problem 5: Can you put the 12 different pentominos together to form a 3 x 20 rectangle? (The 12 pentominos are listed on a separate sheet. You are allowed to rotate them or turn them over, if that helps!)

Can you find two different solutions?



Problem 6: Can you put the 12 different pentominos together to form two separate 5 x 6 rectangles?





References

I. M. AIGNER AND G. M. ZIEGLER, *Proofs from THE BOOK*, Springer-Verlag, Heidelberg Berlin, fourth ed., 2009.

II. F. ARDILA AND R. P. STANLEY, *Tilings*, Mathematical Intelligencer, 32 (2010), pp. 32-43. http://math.sfsu.edu/federico/Articles/tilings.pdf.

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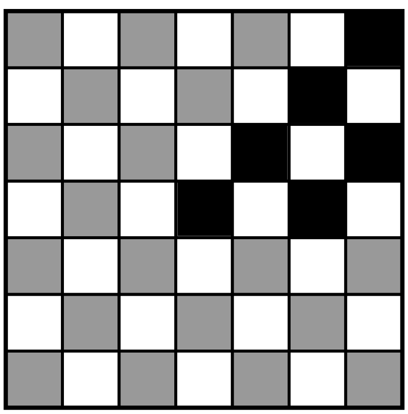
Proofs from THE BOOK: Putting the Pieces Together
Solutions

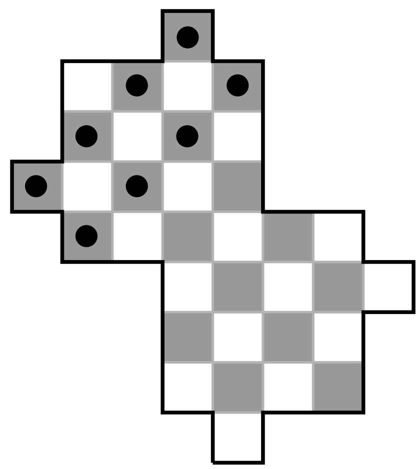
Solution Problem 1: Use the black/white chessboard coloring. Any non-white square can be the “last uncovered square” in the coloring.

Using symmetry, there are still 6 cases to check - see the black squares in the figure.



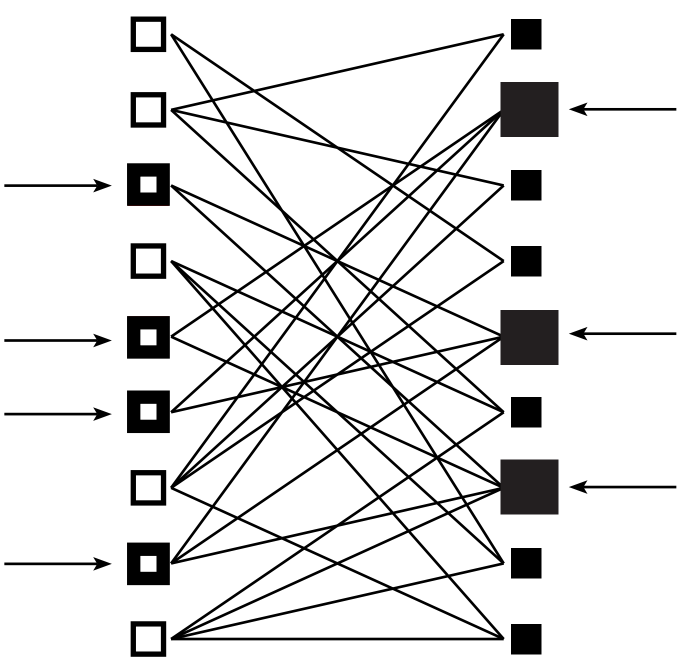
Solution Problem 2: There is no solution.

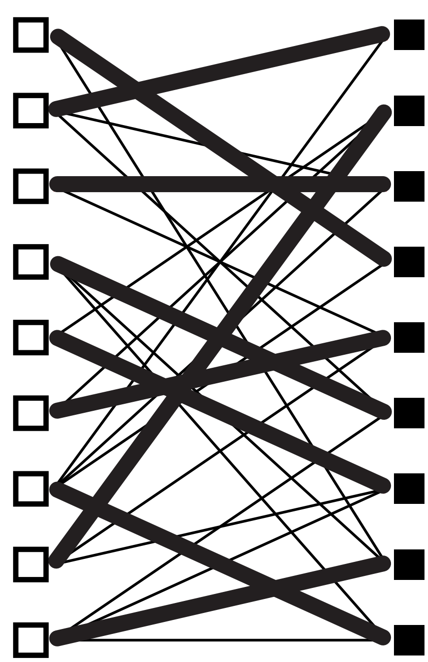
Quick proof (from the “marriage theorem”): The 8 black squares marked by points have only 7 white neighbors.



Solution Problem 3:
In the first situation there is no perfect “marriage arrangement” between the white squares and the black squares: the four thicker-edged white squares only have three different partners (marked by larger dark squares).

In the second situation a solution exists. It can be found by systematically exploring “augmenting chains” - that’s a large research topic in graph theory and in “combinatorial optimization”.





Solution Problem 4:

For T_6, there is no solution; at least three hexagons are left uncovered.

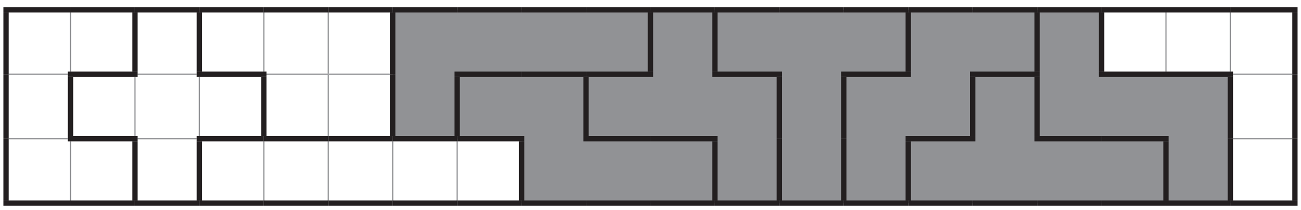
But there is a solution where each hexagon is covered once, except the three hexagons in the middle are covered exactly three times. Thus there is a “signed” solution.

For T_7, there is no solution, as the number of hexagons is not divisible by 3. But there is a nice solution where only the center hexagon is not covered.

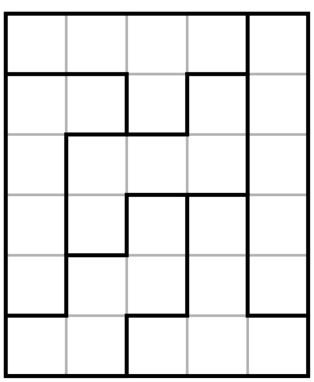
For T_8, again there is no solution, but a signed solution, as for T_6.

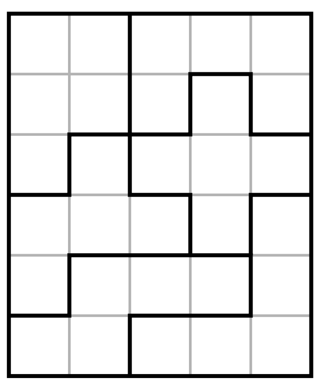
For T_9, there is a solution!

Solution Problem 5:



Solution Problem 6:





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12 Pentominos

Cut along the outside edges of each of the shapes below to create your own set of pentominos.