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Mathscapes
April 2026

Title

We're All In This Together

Real world event

Earth Day

Problem

Four fishing families share the same body of water. Each family sends out some (possibly different) number of fishing boats. If S is the total number of boats sent out by all families, then each boat catches 100 − S fish. (The formula is inspired by the Gordon-Schaefer model for fishery economics.)

To simplify analysis, assume that boats and fish can be fractional, in the sense that if the families send out A, B, C, and D many boats each (where they can be non-integers), then in terms of the sum S = A + B + C + D, the first family catches a total of A × (100 − S) fish, etc.

Question 1: If the families cooperate and share the proceeds, what is the largest total number of fish they can catch?

Question 2: Suppose each of the 4 families independently decides how many boats to send, maximizing their own family's catch. A (pure) Nash equilibrium is a list of 4 numbers (possibly the same), specifying how many boats each family sends, which satisfies the property that if any one family independently changes their number of boats, that family would not catch more fish. It turns out there is a unique such equilibrium. How many total fish do the 4 families catch in that equilibrium?

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Solution

For Question 1, we need to maximize the total number of fish caught, which is:

$A \times (100 - S) + B \times (100 - S) + C \times (100 - S) + D \times (100 - S) = S \times (100 - S)$.

This is the same problem as figuring out the maximum product of two positive numbers which sum to 100. That happens when they are equal. So the optimum is to send 50 fishing boats altogether, catching a total of $50 \times (100 - 50) = 2500$ fish.

For Question 2, the equilibrium condition implies that A is a value of T which maximizes:

$T \times (100 - T - B - C - D)$.

This is the product of 2 numbers which sum to $100 - B - C - D$. Importantly, even as T varies, that sum remains constant. So, as before, the product is maximized when the summands are equal, which means $A = 100 - A - B - C - D = 100 - S$.

The same argument then gives that B, C, and D are all also equal to $100 - S$, hence equal to each other. Thus $S = 4A$, and:

$A = 100 - 4A$,

which gives $5A = 100$, or $A = 20$. There is therefore only one equilibrium candidate, which is $A = B = C = D = 20$. A quick check shows that this is really an equilibrium, because $T \times (100 - T - 20 - 20 - 20) = T \times (40 - T)$ is indeed maximized when $T = 20$.

That would be a total of $20 + 20 + 20 + 20 = 80$ boats, catching a total of $80 \times (100 - 80) = 1600$ fish, which is worse than if they cooperated.