MOMATH
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Mathscapes
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Title

Risk vs. Reward

Real world event

Winter Olympics

Problem

The Olympics are a fascinating showcase of people probing the limits of human potential. Sometimes, when going for the gold, it's winner-takes-all. And often, athletes and their coaches are faced with a big question: is it better to overstretch with a more risky maneuver, which has some chance of a stronger outcome, but also some chance of a weaker outcome? Or is it better to play it safe for a sure, but moderate result?

Perhaps you also have encountered similar situations in real life.

Consider this model, which is simple enough to analyze, but has a subtle and surprising analysis.

Players A and B are in a 1 vs. 1 final matchup which will determine who gets the gold medal.

Player A has two levels of performance to select between: (1) guaranteed 100 points, or (2) randomly 25% chance of 110 points and 75% chance of 90 points.

Player B has two levels of performance to select between: (1) guaranteed 105 points, or (2) randomly 25% chance of 115 points and 75% chance of 95 points.

They play one time to earn these points. Whomever has more points wins the matchup.

It turns out that there is a real number p which satisfies both:

- There is a strategy for Player A that guarantees Player A a chance of winning that is greater than or equal to p, no matter how Player B plays; and
- There is a strategy for Player B that guarantees Player A a chance of winning that is less than or equal to p, no matter how Player A plays.

What is p? What are the strategies?

Hint: A player's strategy does not have to be a definite choice; it may involve additional randomness. One possible (non-optimal) strategy for Player A is to first flip a fair coin, and if it turns up heads, go for performance level (1), and it turns up tails, go for performance level (2). This would give Player A a 37.5% chance of getting 90 points, a 50% chance of getting 100 points, and a 12.5% chance of getting 110 points.

Solution

This problem shows the value of *mixed strategies*, which are where players use additional randomness to decide which performance level to select. Indeed, a strategy for Player A is just a real number q which is the probability of Player A choosing performance level (1). A strategy for Player B is just a real number r which is the probability of Player B choosing performance level (1).

Let's start by figuring out a good choice of q (for Player A) which guarantees the highest possible win probability, no matter what Player B chooses for r.

The "no matter what" is a useful concept. It means that Player B could even know what Player A was going to use for q. That simplifies the analysis.

In terms of q, if Player B chooses performance level (1), then the probability that Player A wins is $(1 – q)(1/4)$. And if Player B chooses performance level (2), then the probability that Player A wins is $(3/4)(q + (1 – q)(1/4))$. Player B would choose whichever one would give a lower win probability for Player A. So Player A's win probability with a given choice of q against an optimally-playing Player B is:

$\min\{ (1 – q)(1/4), (3/4)(q + (1 – q)(1/4)) \}$.

If you graph this in the range $0 \le q \le 1$, you see that is two line segments, first increasing, then decreasing. So the maximum possible value is when the two arguments of the minimum are equal. Solving:

$(1 – q)(1/4) = (3/4)(q + (1 – q)(1/4))$

gives $q = 1/13$. And $(1 – 1/13)(1/4) = 3/13$.

So that choice of $q = 1/13$ for Player A gives a win probability which is always at least 3/13, no matter what Player B chooses for r.

For the second part of the question, consider the world from Player B's point of view. We must find the optimal choice of r so that no matter what Player A chooses for q (even if Player A already knew exactly what Player B was choosing for r), the probability of Player A winning is minimized.

We use similar analysis. If Player B is using r, then if Player A uses performance level (1), Player A's win probability is $(1 – r)(3/4)$. And if Player A uses performance level (2), then Player A's win probability is $(1/4)(r + (1 – r)(3/4))$. Player A would attempt to maximize between these two options, so the probability of Player A winning would be

$\max\{ (1 – r)(3/4), (1/4)(r + (1 – r)(3/4)) \}$.

If you graph this over the range $0 \leq r \leq 1$, the graph is two line segments, first decreasing and then increasing. So the minimum value (best choice for Player B) is when the two arguments are equal. Solving:

$(1 - r)(3/4) = (1/4)(r + (1 - r)(3/4))$

gives $r = 9/13$. And $(1 - 9/13)(3/4) = 3/13$.

So that choice of $r = 9/13$ for Player B gives a win probability for Player A which is always at most 3/13, no matter what Player A chooses for q.

Since both analyses produce the same value 3/13, that is the value of p.

Therefore, perhaps surprisingly, it turns out that whenever you are faced with a situation like this, your optimal strategy might be to introduce some additional randomness yourself. Don't always take the guaranteed route. Don't always take the risky route. Good luck in real life!