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Mathscapes
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Title

Risk vs. Reward

Real world event

Winter Olympics

Problem

The Olympics are a fascinating showcase of people probing the limits of human potential. Sometimes, when going for the gold, it's winner-takes-all. And often, athletes and their coaches are faced with a big question: is it better to overstretch with a more risky maneuver, which has some chance of a stronger outcome, but also some chance of a weaker outcome? Or is it better to play it safe for a sure, but moderate result?

Perhaps you also have encountered similar situations in real life.

Consider this model, which is simple enough to analyze, but has a subtle and surprising analysis.

Players A and B are in a 1 vs. 1 final matchup which will determine who gets the gold medal.

Player A has two levels of performance to select between: (1) guaranteed 100 points, or (2) randomly 25% chance of 110 points and 75% chance of 90 points.

Player B has two levels of performance to select between: (1) guaranteed 105 points, or (2) randomly 25% chance of 115 points and 75% chance of 95 points.

They play one time to earn these points. Whomever has more points wins the matchup.

It turns out that there is a real number p which satisfies both:

- There is a strategy for Player A that guarantees Player A a chance of winning that is greater than or equal to p, no matter how Player B plays; and
- There is a strategy for Player B that guarantees Player A a chance of winning that is less than or equal to p, no matter how Player A plays.

What is p? What are the strategies?

Hint: A player's strategy does not have to be a definite choice; it may involve additional randomness. One possible (non-optimal) strategy for Player A is to first flip a fair coin, and if it turns up heads, go for performance level (1), and it turns up tails, go for performance level (2). This would give Player A a 37.5% chance of getting 90 points, a 50% chance of getting 100 points, and a 12.5% chance of getting 110 points.