MOMATH

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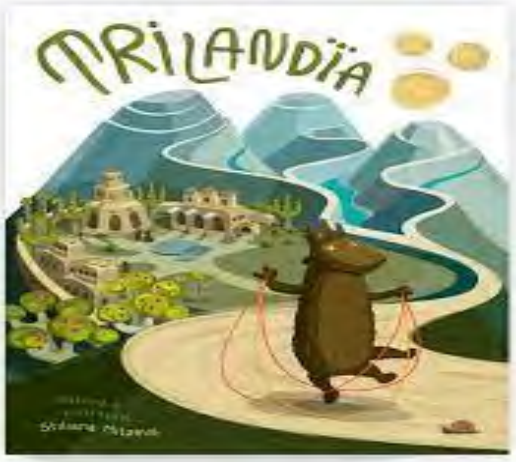
Journey to Trilandia

Shefali Nanavati

Grade 6

Journey to Trilandia: Exploring Base-3 Number System



Shefali Nanavati

Lesson Plan
Grade 6

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Lesson Overview

This is a fun, interactive lesson that can be used to teach different concepts such as number systems, exponents, place value, and algebraic reasoning. I have used this lesson in Grade 6 as an introduction to algebraic thinking, focusing on evaluating expressions through substitution. It has interdisciplinary connections with history and art, as it lends students to learn about historical number systems and their symbols. Students "travel" to a far away land known as Trilandia. During the journey, students learn how to trade in a base-3 number system and convert that value to our base-10 number system. Once students feel comfortable trading in base-3, they can choose to increase the stakes by either using a larger sided die or compete against one another to see who leaves Trilandia with the most coins. Using critical thinking and problem solving techniques, students deepen their understanding of what place value means by engaging in hands-on approaches. Students discover patterns in a base-3 place value system that can be transferred to other base systems. Throughout the lesson, students consider the following:

- How can you convert a number in base-3 to its equivalent value in base-10?
- Can you explain why 221 =25?
- Why does the number 3 never appear in the base-3 number system?

Lesson Goals:

- Use knowledge of base-10 place value system to explore other place value systems
- Write numbers using a base-3 number system
- Convert values from base-3 to base-10 by substituting and solving variables with given values
- Justify that 221 = 25

Background Knowledge and Context:

This lesson can be used to support various concepts in a math curriculum. It can fit into number sense as it deepens student knowledge of place value. I use it as a jumping off point to begin our algebra unit, as it organically uses variables and leads students to evaluate expressions through substitution. The following background knowledge may be helpful as students engage in the activity:

- Our place value system (1, 10ths, 100ths…) increases in increments of powers of 10
- An exponent is the number of times the base number is multiplied.
- A loss in value is represented by a negative
- Our monetary system trades or equations such as the following:
  5 pennies = 1 nickel, 2 nickels= 1 dime, 10 dimes= 1 dollar, etc…

The Common Core Standards for Mathematics

Content Standards:

CCSS.5.NBT.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

CCSS.5.NBT.2: Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.

CCSS.6.EE4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.

CCSS.6.EE.6: Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

CCSS.7.EE.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

Practice Standards:

CCSS.MP1: Make sense of problems and persevere in solving them.
CCSS.MP2: Reason abstractly and quantitatively.

Activity Details

Materials:

1. Various Sided Dice (standard, 8-sided dice, 12-sided dice, 20-sided dice)

2. Action Cube: A standard dice with + and - sides

3. Paper Plates to represent the “bank”

4. Colored Chips or Tiles. Preferred colors: yellow, blue, green, red, black

5. Worksheets: Wallet, recording sheet, reflection questions (worksheets at end of lesson plan).

Here is your Trilandia Wallet. Use this to explore trading in base-3. Record what is in your wallet on the recording sheet. Remember, you don't want a heavy wallet!

3^4 = 81	3^3 = 27	3^2 = 9	3^1 = 3	3^0 = 1

Key:
3y=1b
3b= 1g
3g= 1r
3r = 1B

Name: _____

Warm up:

Tri-Landia Recording Sheet

1. Roll action cube. Roll number cube.
2. Everyone makes the trade. You must make all trades possible to have a "light" wallet.
3. Record your final answer as an addition expression. Base-3 and Base-10.

ACTION	WALLET	BASE-3	BASE-10
/	1g+1b+1y	111	13

Tri-Landia Recording Sheet **Name:** _____

1. Roll action cube. Roll number cube.
2. Everyone makes the trade. You must make all trades possible to have a "light" wallet.
3. Record your final answer as an addition expression. Base-3 and Base-10.

ACTION	WALLET	BASE-3	BASE-10 Value
/	1g+1b+1y	111	13

Name:_____

Please take a moment to reflect on your experience in Trilandia.

Reflection Questions:

1. What was challenging/interesting about today's activity?

2. Can you now explain how 25 = 221

3. Why does a "3" never appear in the base-3? How does that connect to 10 in base-10?

4. Did you end up in the negatives? If so, how did you deal with debt?

5. Do you have any questions/comments/wonderings?

Preparation: Time: 15 minutes

- Class set of copies of "wallet" and recording sheet
- Prepare quart size ziplock bags for each pair of students containing the following:
    - one of each dice
    - 15-20 chip/tile of each color
    - a paper plate
- Project or post directions for students to reference as they play.
- Project or post the reflection questions or create as a worksheet to generate written responses

Depending on the class size, it is helpful to demonstrate the activity visually, so the class can "warm up" as a whole group to trading in base-3. I use my SmartBoard and document camera to demonstrate activity for the warm up.

Activity Description

Steps: Time: Approximately 50 minutes

1. Review our base 10 number system. Point out that each place value is a power of 10. So the base number is 10 and the exponent represents the place value. Hence the name base-10 number system. (5 minutes)

2. Engage students into the lesson by telling them they are about to embark on a journey. Have them visualize a far-away land called Trilandia. Students usually get the idea and finish these sentences about life in Trilandia for you: “In Trilandia people look different. Instead of two eyes, they have (3). Instead of bicycles, they have (tricycles). Books are written in (trilogies), no twins just (triplets), and they have a base-3 number system, etc… (2 minutes)

3. Show Trilandia’s base-3 place value by using the wallet worksheet. Explain Trilandian coins.Here are the coins values: each yellow =3^0 or 1, blue= 3^1 or 3, green=3^3 or 9, red= 3^4 or 27, and black=3^5 or 81. (3 minutes)

   Here are the colors and trading Rules: 3 yellow = 1 blue, 3 blue=1green, 3 green=1red, 3 red=1 Black

4. Make the wallet promise with your class: “I promise to only touch my wallet, not my partners. I promise to make all the possible trades and not have a “heavy wallet”. A heavy wallet is when there are more than 3 of the same color coin in a place value. That means a trade can be made before continuing play. Tell students that Trilandians are very generous and give you 3 coins as soon as you enter Trilandia: 1 green, 1 blue, and 1 yellow. Students should place all the coins on the paper plate (bank), then take 1 green, 1 blue, and 1 yellow to put in their wallet from the bank. (2 minutes)

5. Lead the class through a warm up. This takes some time as students have to rewire their prior knowledge of trading in 3’s and not 10’s. As a whole class, go through a few rolls of how to trade in Trilandia. Here are the materials to have in front of you for the demo: bank full of coins, wallet, recording sheet, number cube, and action cube. (8 minutes)

   Here are the Game directions:
   a. Roll action cube. Roll number cube.
   b. Everyone makes trade. You have to make all trades possible.

c. Record your final answer as an addition expression (on recording page)
d. Substitute and solve for its value in base-10. (What is the value of yellow coins in your wallet?)*
*For example: 221 in base-3 is 2(9) + 2(3) + 1(1) = 25 in base-10

6. Students can continue to practice with their partner and check to make sure their wallets look the same before they record and convert on the recording sheet. When they finish the warm up page, partners can choose to start over and work together or do their own rolls and see who ends up with the most Trilandian coins at the end of the time. (20 minutes)

Anticipated Student Actions:

- How to deal with debt. One idea that comes up organically with students is to either draw a line in their wallet to represent negative values or put those coins outside the wallet. That way they can keep track of their debt value until they get out of debt (hopefully)
- What happens when you get 3 Black coins? This could happen if students choose to use different sized dice, especially the 20-sided die. Students can create the next Trilandian coin. It is worth a lot! (3^6 = 243).

Reflection Questions: (10 minutes)

1. What was challenging/interesting about today's activity?
2. Can you now explain how 25 = 221 can be a true statement?
3. Why does a "3" never appear in the base-3? How does that connect to 10 in base-10?
4. Did you end up in the negatives? If so, how did you deal with debt?
5. Do you have any questions/comments/wonderings?

Variations:

1. Use a different sided dice to increase the stakes.
2. Explore with a base-4 or base-5 (or other) number system. How would that change your "wallet". How would that affect your trades? What is different about it?

Extensions:

1. History and Art connection: Look into historical number systems, such as Babylonian or Mayan or binary. Discuss what base systems were used. Students can create symbols to represent a different number system that is not base 10, referring to the patterns seen in the Mayan or Babylonian symbols.

2. Create a new coin to represent 3 Black coins and beyond. Have students find the value of those coins as you increase place value in this base-3 system.
3. Optional assessment to check for understanding could be to have students convert between base-10 and base-3 numbers. For example: What does 52 look like in base-3? Answer: 52 = 1221

Acknowledgement

I was introduced to this activity at an Algebraic Reasoning Workshop I attended as a teacher in NYC over 10 years ago. I cannot remember the exact name of the workshop nor the name of the presenter, but I know that I was very inspired. I enjoyed it so much and was enthusiastic to use the idea in my 6th grade math classes. The presenter was very supportive and gave me some of the materials so I could model the lesson in my class. I have since adapted the lesson and tools to make it work in my classroom throughout the years. I am very grateful to have attended the workshop with a supportive and creative presenter, as it has been one of my favorite lessons that I look forward to teaching.

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