Edition 1

December 2019

Math Musings

A publication of the Math Club
Pathways School Gurgaon

Message from School Director and DP Principal.

Greetings!

I am delighted by this wonderful initiative by our student Rohan Jha of DP-1. This magazine, a completely student driven initiative will provide an insight and information on the importance and use of mathematics in our daily lives. A platform which provides a blend of interesting nuggets of information, inspirational news in the area of Mathematics, puzzles and games, the newsletter will certainly pique the curiosity and talent of all alike. Heartiest Congratulations to Rohan and wishing you every success in this specialized and innovative venture.

- **Captain Rohit Sen Bajaj (School Director)**

The magic of numbers eludes many. I am delighted that within the DP space are minds that are fascinated by this elusive world. As Bertrand Russell said, "*The pure Mathematician, like the musician, is a free creator of his world of ordered beauty*. " Without doubt, this team will demystify, intrigue and find the magic charm to engage their peers with mathematics. Good luck!

- **Miss Sangeeta K. Nag (Senior School Principal)**

What is Mathematics?

Mathematics is an expression of the human mind that reflects the active will,

The contemplative reason, and the desire for aesthetic perfection.

"Courant and Robbins"

Prof. Eknath Ghate in his talk on 16th Aug 2019 at Pathways School, Gurgaon,India.

Message from the Founder & President of the Math club

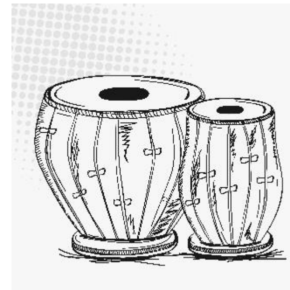
Welcome to the inaugural edition of Math Musings. We were honored and privileged to have Prof Eknath Ghate, TIFR and among the most prominent mathematician in India give the inaugural talk on 16th August, 2019. The title of the talk was The Tau of Ramanujan. In this magazine, we will write thoughtful and well researched articles highlighting the beauty and elegance of mathematics.

- Rohan Jha

CONTENT:

MATH IN OUR LIVES

What has ancient Indian music, nature, and beauty got to do with math!

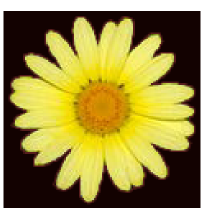


It may come as a surprise to many that in ancient India, poetry was not merely an art form, but was a sophisticated mathematical concept and had to pass the test of Sanskrit shlokas. There was the short syllable (Laghu) which needed 1 beat and the long syllable (Guru) which required 2 beats. Musicians like Tabla players use a similar concept where they call the short syllable "Dhin" and the long syllable "Dha".

Poets had to determine how many beats per shlokas they needed, and the combinations of short beats and long beats they can use. Hemachandra in 1650 AD suggested an algorithm:

Write 1 and 2 and each of the subsequent terms as sum of the preceding two terms.
1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946…………………………………

Suppose the poets had to fill in 8 beats for a shloka. Then, all possible combinations of long and short beats amount to 34 ways which is the 8^{Th} term of the **Hemachandra** sequence! Two hundred years later this came to be known as the Fibonacci series -in the west, popularized by books like **Da Vinci code** by Dan brown most recently.
In nature, many of the flowers have petals which correspond to the Hemachandra sequence. Here is a daisy with 21 petals.



However, this is not the end of the story. When you keep taking the ratios of two terms of the Hemachandra series, higher terms converge towards 1.618034……. which is known as the **golden ratio**. For example, 10964/6765=1.62069. Many artists believe that golden ratio makes the most beautiful shape. Does the rectangle on lower left look beautiful and pleasing to you?

The Hemachandra sequence and the golden ratio keep coming up in important places in advanced mathematics: like partition functions, and recurrence mathematics……

Perhaps all of us at times should stop and admire the beauty and elegance of mathematics, and pay tribute to Hemachandra!

The Mathematics of Magic Tricks

Magicians are known for their amazing tricks and sleight of hand. Mathematicians on the other hand, use vigorous and sound logic to seek the ultimate truth about numbers. When the two collide, it often leads to scintillating magic! It would no doubt surprise many of you to know that some of the top magicians in the world are also very good mathematicians! Here is a magic trick that has no conjuring but based on pure mathematics which we learn in school.

The Trick!

- Take a deck of 27 playing cards

- Without you seeing any of the cards, spread them and ask the audience to pick a card and remember it. You of course don't know what that card is! But let's say it is the 10 of hearts.

- You then shuffle the deck several times. Ask the audience to pick a number between 1 and 27. Suppose they tell you the number to be 10.

- The deck is held face down and dealt into three face-up piles of 9 cards each by placing a card on each of the piles as we deal face up. Ask the audience to identify the pile in which your card is placed.

- Then, taking each of the pile and placing then in top, middle and bottom in some order repeat the previous step two more times- altogether this is done 3 times.

- Then count the 10th card (Audience has chosen the number 10) and then turn it over – it is 10 of hearts! Amazing!

Why does this magic trick work!

To understand why the trick works, we have to review some basic mathematics. In particular, we have to focus on the ternary counting system. We are used to decimal counting system of base 10 – the digits go from 0 to 9. In the ternary system of base 3, the digits go from 0 to 2. Let us first list the ternary system of numbers from 0 through 27.

Decimal	Ternary	Decimal	Ternary	Decimal	Ternary	Decimal	Ternary
0	000	7	021	14	112	21	210
1	001	8	022	15	120	22	211
2	002	9	100	16	121	23	212
3	010	10	101	17	122	24	220
4	011	11	102	18	200	25	221
5	012	12	110	19	201	26	222
6	020	13	111	20	202	27	1000

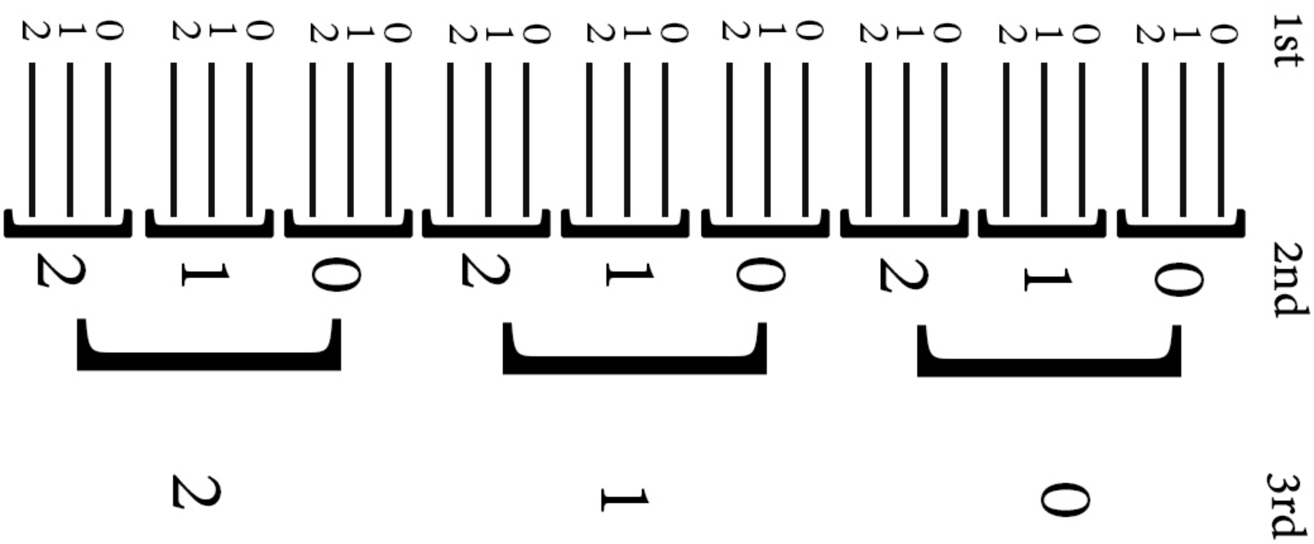
The last digit of a ternary number indicates units, the next to last digit stands for "threes", the third from last stands for "nines" and so on. Thus, to translate the ternary number 100 into our decimal system you have only to multiply the first digit by 9, add the product of the second digits times 3, and then add the last digit. In this case, 9 plus 0 plus 0 equals 9- the decimal equivalent of the ternary number 100.

But how does it apply to our magic trick?

Since the audience had chosen 10 from the numbers 1 through 27, we must bring 9 cards above it. As we have just seen, the ternary equivalent of 9 is 100. Taking these digits in reverse order gives us 001. This tells us how to pick up the piles each time - 0 standing for top, 1 for middle, 2 for bottom. In other words, our first pickup places the pile containing the chosen card on top, the second assembly does the same and the final assembly places the pile in the middle. The card will then be 10th from the top!

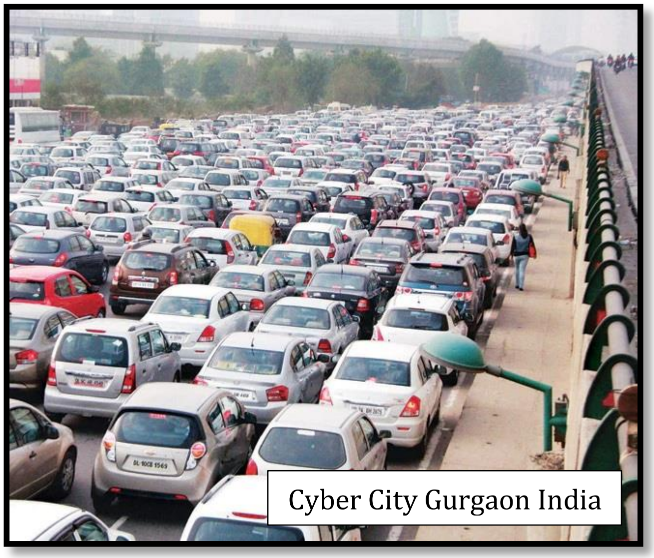
Fear not if you still are having trouble understanding the trick!

A visual way of understanding why the trick works is shown below. The first assembly isolates the chosen card to 9, the second to 3 and the final assembly to 1 in the ternary counting system!



The Mathematics of Traffic Jams!

We have all experienced Traffic jams! It has caused us to miss flights, appointments and even start of movies! Economically, it has an adverse effects as well. Many of the businesses suffer as they are not able to deliver their products on time. Sometimes, it has serious consequences as well, as medical help gets delayed for patients.

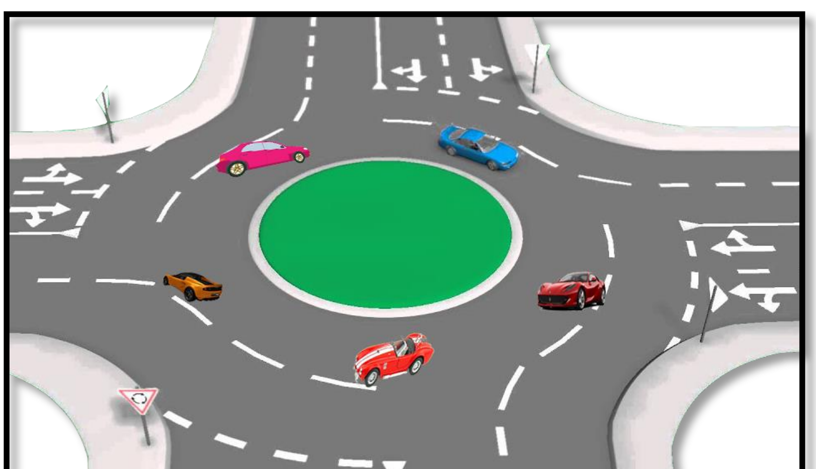


Cyber City Gurgaon India

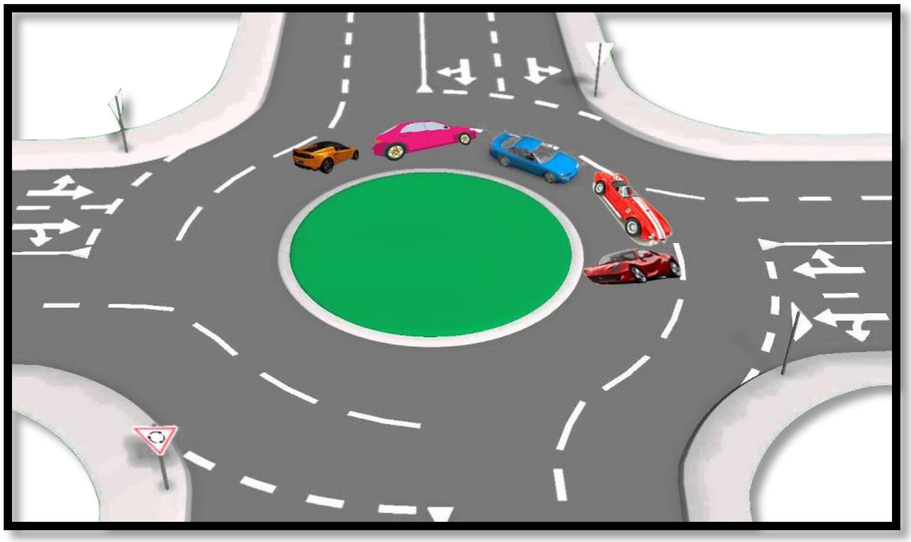
Why do Traffic jams happen?

When we are caught in a traffic jam, our first instinct is that there has been an accident. However, we get surprised as we realize that there was no accident or obvious reasons for the delay!

A simple experiment illustrates this effect. Suppose we ask five drivers to drive their cars in a circle maintaining a constant speed of 20 km/hr.

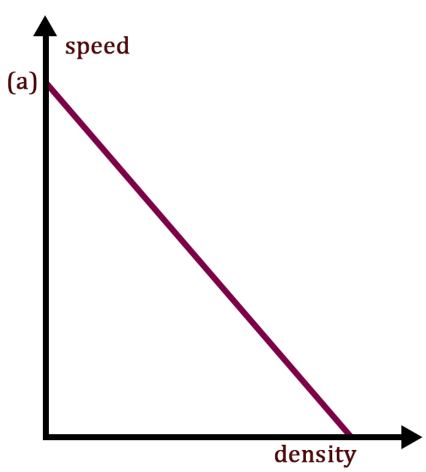


After some time, we notice that all the five cars are clustered together! The reason being that human beings find it impossible to maintain a constant speed. Some drive faster and some drive slower, giving rise to a **phantom traffic jam**. In fact, phantom traffic jams are the most frequent type of traffic jams!

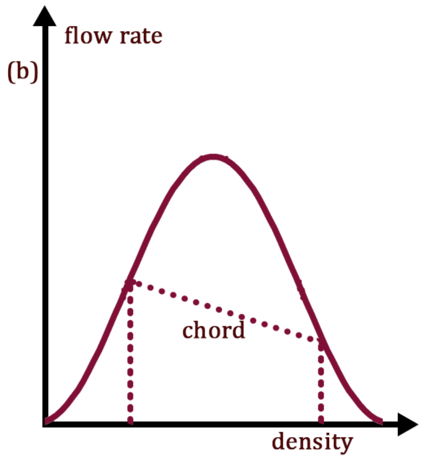


Mathematics can help us greatly with the problem of Traffic Jams!

One way mathematicians model the traffic is to treat it like a gas, where each car is modelled as a molecule of gas. This enables mathematicians to use equations about how gas flows down pipes. The graph below shows how speed and density are related. When cars are densely packed in on a highway, their speed is low, as cars drive slower in order not to crash into one another. When on the other hand the density of cars is low, then cars can drive faster as there aren't many cars around to get in their way.



The next graph shows how flow rate and density are related. (Flow rate is number of cars passing a given point in a given time.) We want to maximize the flow rate, as this will mean more cars will be able to travel down the highway. The relationship between flow rate and density is a bell shaped graph.



When the density of cars is low, then flow rate is low as few cars are travelling down the highway. When density of cars is very high then flow rate is also low as traffic jams will develop and few cars will be flowing down the highway.

A potential solution to Phantom Traffic Jams

The graph of flow rate versus density gives us a clue. If the authorities study the traffic flow and based on flow rate and density during peak hours, reduce/increase the speed limit so that we are close to maximum value of the flow rate – density bell shaped graph, then the traffic will flow at a much faster rate! It is to be noted that they have to ensure that the density does not increase too much as the flow rate will again dip.

This shows that reducing traffic jams does not always involve major infrastructure expense but just implementation of smart mathematics!

WHAT WE'VE BEEN UP TO!

Math Club's Inaugural talk by Prof Eknath Ghate - 16th August 2019

The inaugural talk of the Math club was given by Prof Eknath Ghate of the renowned institution TIFR (Tata institute of Fundamental Research), Mumbai. He did his undergraduate studies from University of Pennsylvania and his PhD from University of California, Los Angeles. He has been the recipient of the prestigious Bhatnagar award in 2013 and is also a fellow of Indian Academy of Sciences.

The title of the presentation was the "Tau of Ramanujan" and his enchanting talk took us through an amazing journey of the math behind Sudoku and some of the grand ideas of great mathematicians like Euler, Gauss, ending with the Tau function of the Indian genius Ramanujan. The talk was very interactive and all the students of DP1 and DP2 enjoyed it greatly. The session ended with some of the DP2 students having a conversation with Prof Ghate about his mathematical journey

Math Club Visits NGO!

Among the founding charter of the math club is to teach real life math skills to underprivileged kids in an NGO. Therefore, we are pleased to announce our partnership with Yashpal Foundation, an NGO located in Vasant Kunj, New Delhi. The sad part is that many of the kids are forced to leave the NGO as soon as their family feel they are able to get some menial work -despite the efforts of the Foundation to dissuade such actions.

A few of our members visited recently. The team consisted of: Rohan Jha, Ayaan Goel, Viraj Choudhary, Tegh Kalsey, and Rishi Tayal. The kids at the NGO were of different age groups, ranging from 4th grade all the way to 10th. We spent the initial part of the session getting to know the kids and inquiring about their math knowledge.

The kids were very enthusiastic and delighted to have us and soon after arranging for a blackboard and chalk we started explaining the Pythagoras theorem -among other things. The session was highly interactive and we were asked a lot of questions! This session helped us to plan the future set of topics to cover so that they learn math beyond basic arithmetic.

Overall, interacting with a group of kids with a strong desire to learn was extremely satisfying and pleasing. We look forward to conducting many more visits in the near future. Thank you, Yashpal Foundation for your wonderful contribution to the welfare of these kids. We are with you always!

MATHEMATICAL DISCOVERY CAN BE MADE IN SCHOOL!

Chika Ofili, a Nigerian boy, 12-year-old living in London just discovered a new Formula for divisibility by 7 in maths.

Multiply the last digit by 5 and add it to the remaining numbers.

For Example 532

53+2 x 5 =63 which is a multiple of 7, So 532 is divisible by 7.

Chika Ofili has been presented with a special Recognition Award for making a new discovery in mathematics

Here is a simple Proof:-

If x+5b=0 mod 7 then multiplying by 10 we get

10x+50b=0 mod 7.

But 50 is just 1 mod 7 so we get 10x+b=0 mod 7 as an equivalent statement.

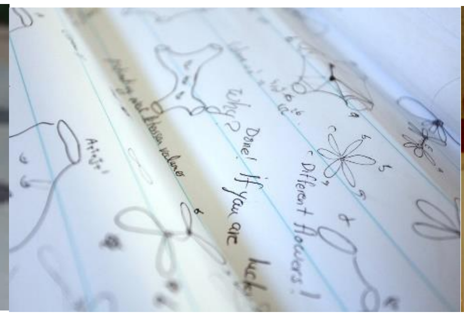
The spectacular Persi Diaconis!

One of the most famous examples of magicians who is also a world-renowned mathematician is Prof. Diaconis at Stanford University. As the story goes, at the age of 14 he ran away from home to work under the magician, Dai Vernon. A few years later he was admitted to Harvard University's graduate statistics program on the strength of a recommendation letter from the famed mathematics writer Martin Gardner who was impressed with his card tricks.

Now a professor of mathematics and statistics at Stanford University, Professor Diaconis has employed his intuition about cards in a wide range of situations. Once, for example, he helped decode messages passed between inmates at a California state prison by using small random "shuffles" to gradually improve a decryption key. He has also analyzed Bose-Einstein condensation — in which a collection of ultra-cold atoms coalesces into a single big atom — by imagining the atoms as rows of cards moving around. It seems he has made original creative paintings in the canvas of mathematics pursuing his passion for magic!

Inspirational story of Maryam Mirzakhani

Maryam Mirzakhani was an Iranian mathematician and a professor of mathematics at Stanford University. She was the first woman to be honored with the quadrennial (given every four years) Fields Medal, which she won in 2014, often equated in stature with the Nobel Prize.


To solve problems, Mirzakhani would draw doodles on sheets of paper, and write mathematical formulas around the drawings. Her daughter described her work as "painting".

Her recent work investigated a mathematical challenge that physicists have struggled with for a century: the trajectory of a billiard ball around a polygonal table. The investigation into this seemingly simple action led to a 200-page paper (published in 2013) which, was hailed as “the beginning of a new era” in mathematics and “a titanic work.”

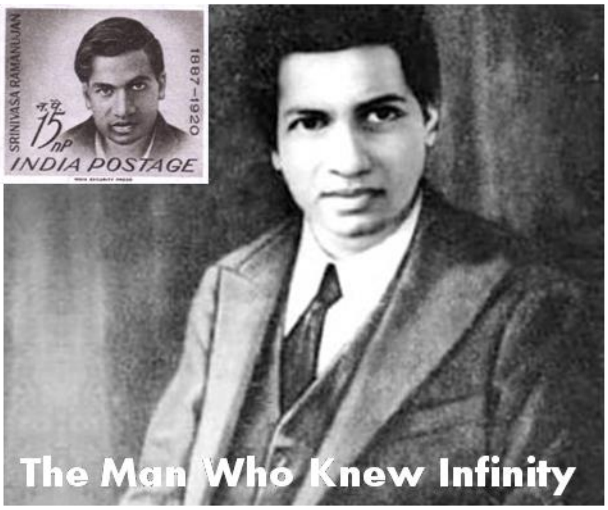
“You have to spend some energy and effort to see the beauty of math,” she once told a reporter.

Her research topics include Topology and study of dynamical systems. Topology is the mathematical study of the properties that are preserved through deformations, twisting, and stretching of objects, that is, remain same even after deformations, for example, A circle is topologically equivalent to an ellipse, into which it can be deformed by stretching. Her work was highly theoretical in nature, but it could have impacts on the theoretical physics of how the universe came into existence.

On July 14, 2017, Mirzakhani died of Breast Cancer at the age of 40, which she had struggled with since 2013. Her legacy would no doubt continue to inspire women for decades if not centuries to come.

“Maryam is gone far too soon, but her impact will live on for the thousands of women she inspired to pursue math and science,” said Stanford President Marc Tessier-Lavigne.

Celebrating Ramanujan's Birthday on Dec 22nd



Srinivasa Ramanujan is the greatest mathematician to come from India. He was a self-taught mathematician. Hindered by poverty and ill-health, his original work has considerably enriched the fields of sciences such as number theory and, more recently, physics and he tragically died at the tender age of 33 (1887 -1920).

Although a math prodigy, he failed to get a college degree, as he did not clear his fine arts courses. However, he wrote to Professor G.H Hardy while working as an accounting clerk with the Madras Port Trust who accepted him at Cambridge in April 1914, seeing his prodigious talent for mathematics. In 1916, he got the equivalent of a Ph.D. and in 1918 he became the first Indian to be elected as a Fellow of the British Royal Society. In India, we celebrate National mathematics day on 22nd December in honor of his birthday.

Once when Ramanujan was in the hospital, Professor Hardy came to visit him and mentioned that the Taxicab he came in had the boring number 1729 and hoped that it was not a bad omen. Ramanujan immediately replied that it was, in fact, a very interesting number as it was the smallest number that could be expressed by the sum of two cubes in two different ways.

$$\mathbf{1729 = 1^3 + 12^3 = 10^3 + 9^3}$$

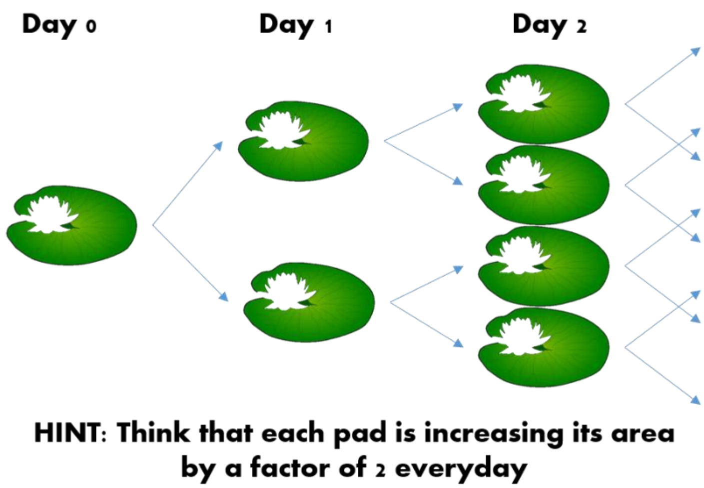
It is now known as the Taxicab number. Here are some other examples of this kind of numbers

$$\mathbf{4104 = 16^3 + 2^3 = 15^3 + 9^3}$$

$$\mathbf{13882 = 24^3 + 2^3 = 20^3 + 18^3}$$

SOLVE THE PUZZLE!

You start with a single lily pad sitting on an otherwise empty pond. You are told that the surface area of the single lily pad doubles every day and that it takes 24 days for the single lily pad to cover the surface of the pond.



If instead of one lily pad you start with eight lily pads (each identical to the single lily pad), how many days will it take for the surface of the pond to become covered?

Answer: If you can figure out the relationship between **8** lily pads and one lily pad you will get the answer! Since one Lily pad doubles every day, after **3** days it will be equivalent to starting with **8** lily pads! Then since one Lily pad covers the whole pond in **24** days and we are starting after three days for **8** lily pads, the answer is **24-3 = 21 days**!

Answer: 21 day

Become An Instant World Famous Math Celebrity!

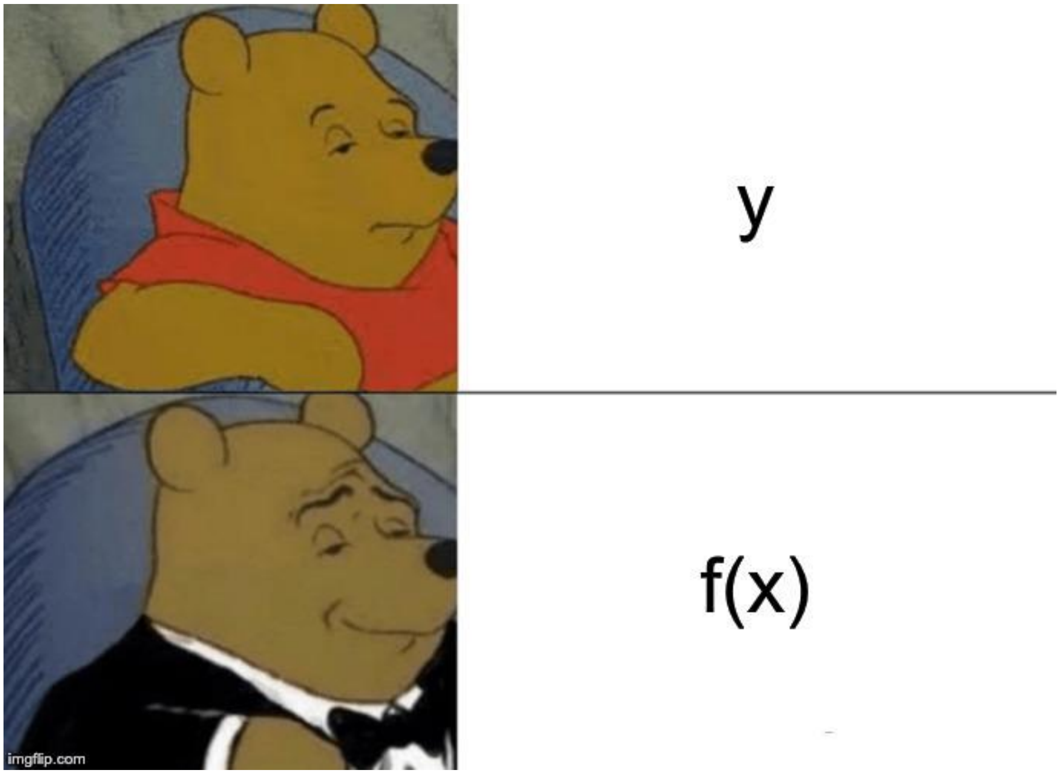
$$\sum_{n=1}^{\infty}\frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \dots\dots\dots = \infty$$

$$\sum_{n=1}^{\infty}\frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \dots\dots\dots = \frac{\pi^2}{6}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^4} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} \dots\dots\dots = \frac{\pi^4}{90}$$

What is the answer to?

$$\sum_{n=1}^{\infty}\frac{1}{n^3} = 1 + \frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} \dots\dots\dots = ?$$



$$y = \frac{log_e\left(\frac{x}{m} - sa\right)}{r^2}$$

$$yr^2 = log_e\left(\frac{x}{m} - sa\right)$$

$$e^{yr^2} = \frac{x}{m} - sa$$

$$me^{yr^2} = x - msa$$

$$me^{rry} = x - mas$$