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THE 2019 ROSENTHAL PRIZE

Innovation and Inspiration in Math Teaching

Dice Auction

Putting Outcomes of the Dice Up for Sale

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Lesson Plan

Grade 7 Probability

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Task at a Glance

In the Dice Auction, every lot up for sale is a possible event when rolling a pair of 6-sided dice (e.g. “A double is rolled” or “The numbers sum to a perfect square”). Groups of students are given a fixed budget with which to purchase lots, and must use their understanding of theoretical and experimental probability (of both simple and compound events) to inform which lots to target and appropriate prices to pay. After the fifteen lots have been auctioned off, a pair of 6-sided dice are rolled twenty times, and students receive a prize each time one of their purchased lots occurs.

Lesson Goals

The Dice Auction asks groups of students to investigate chance processes and develop models to analyze the lots up for sale, but also asks them to assign a reasonable price to each event. By requiring them to pay attention to the market set by their peers at auction, the focus becomes the connections between events: Which are more likely? Which *seem* more likely? What might classmates pay? What price is fair? Where can I land the best deal? Through this, students explore what theoretical probability tells us about expected value, how decisions affect the market’s notion of fair, and where the auction itself generated particularly unexpected results.

Throughout the task, student decision-making is paramount. Students not only use probability models to decide a fair price, those determinations shift the market for subsequent decisions. Therefore, the task asks students to use their formal models of probability in tandem with subjective impressions of the class dynamic to determine what each lot is worth. Because each decision has the potential to shift the next lot’s value, the task highlights that the probability of an event depends on the information you have about an event. Ultimately, the task combines a familiar pair of dice with novel opportunities to make decisions and analyze the mathematical fallout from said decisions.

Students will:

- Calculate the probability of compound events using a two-dice distribution model.
- Compare the favorable outcomes of similar events to determine which event is more likely.
- Use the theoretical probabilities along with subjective information about the class to determine fair market prices for the events.
- Compare experimental results with theoretical expectations and identify and explain possible discrepancies.

Common Core Standards for Mathematical Practice

<u>CCSS.MATH.PRACTICE.MP2</u> *Reason abstractly and quantitatively.*

Students reason abstractly and quantitatively when they apply a dice distribution to their intuitions of likelihood. Students re-contextualize when they make a judgement of a fair price.

<u>CCSS.MATH.PRACTICE.MP3</u> *Construct viable arguments and critique the reasoning of others.*

Students construct arguments when their teams decide on the events to target. Students critique the reasoning of others when they reflect on the fair prices set by their peers at auction.

<u>CCSS.MATH.PRACTICE.MP4</u> *Model with mathematics.*

Students model with mathematics when they use the dice distribution to quantify theoretical possibilities and when these likelihoods are modeled through experimental dice rolls.

<u>CCSS.MATH.PRACTICE.MP6</u> *Attend to precision.*

Students attend to precision when they communicate about the differences between what should happen (in theory), what did happen (through experiment), and the connection between the two.

<u>CCSS.MATH.PRACTICE.MP8</u> *Look for and express regularity in repeated reasoning.*

Students look for repeated reasoning when they recognize that some events may be different in name but identical in frequency (e.g. “a single 1 is rolled” and “a single 3 is rolled”).

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Common Core Standards for Mathematical Content

CCSS.MATH.CONTENT.7.SP.C.5
Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring.

CCSS.MATH.CONTENT.7.SP.C.6
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

CCSS.MATH.CONTENT.7.SP.C.7
Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

CCSS.MATH.CONTENT.7.SP.C.7.A
Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

CCSS.MATH.CONTENT.7.SP.C.8
Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

CCSS.MATH.CONTENT.7.SP.C.8.A
Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

CCSS.MATH.CONTENT.7.SP.C.8.B
Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

CCSS.MATH.CONTENT.4.NF.A.2
Compare two fractions with different numerators and different denominators.

Prerequisite Knowledge

Dice Auction sits at an important point in a student's mathematical education because the standards it addresses are the initial outcomes in the study of probability, which lay the foundation for a strong understanding of probability concepts for years to come. Students have been making determinations of likelihood in their everyday experiences, and these impressions will likely make their way into their justifications. Dice Auction is intended to draw out a student's implicit notions and tether them to more formalized notions of probability. However, students will enter into sophisticated thinking if they have (at least) an initial understanding of the theoretical probability of an event as the number of favorable outcomes divided by the total number of possible outcomes, of experimental probability as the number of successful trials divided by the total number of trials in the experiment, and of the two-dice distribution chart.

The choice to roll the dice twenty times is made intentionally. Students will use the dice distribution chart to calculate many of the theoretical probabilities, resulting in lots typically having probabilities with denominators of 36 (and reducing to denominators like 18, 12, 9, 6, 4—factors of 36). After the dice are rolled twenty times, students can then construct the experimental probability of lots with denominators of 20 (and reducing to denominators like 10, 5, 4, 2—factors of 20). When students are asked to determine a lot that had unexpected rolling results, they may look at a comparison between its theoretical and experimental probability. Comparing fractions with unlike denominators can therefore be considered a pre-requisite skill. Additionally, some of the lots will require an understanding of mathematical definitions and vocabulary: sum, difference, product, prime, odd, multiple, perfect square, consecutive, etc.

Time Required

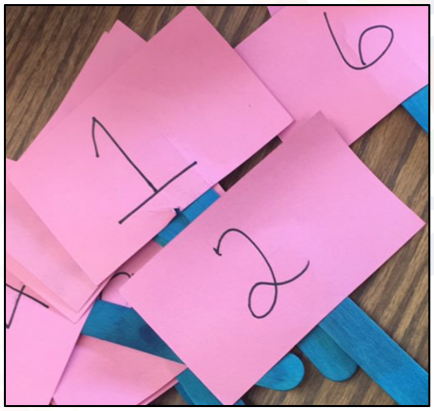
Preparation time:	15 – 30 minutes
Class time:	50 – 60 minutes

Setup

Materials

Dice Auction involves materials that are easy to replicate, readily available in middle schools, or inexpensive to acquire.

- A single pair of standard 6-sided dice.
- Handouts for each student:
  - "Lots for Sale" (and possibly the "Expansion Auction Lots"), "Dice Distribution Charts," "Dice Rolls," and "Personal Reflection."
  - *All five handouts are available at the end of this lesson plan under the "Materials for Copying" section*.
- Auction paddles for each group.
  - Paddles can be made from popsicle sticks and index cards or other easily available materials.



- Small items to be used as prizes.
  - A collection of wrapped candies from the bulk section of your supermarket works well.
- A gavel.
  - While this is optional, it adds to the atmosphere.
- A currency.
  - Each group begins with 100 units of currency. Groups can track this on paper, or you can provide them with physical objects (snap cubes or Base-10 unit cubes work well and may be available in your school.)
  - *If you provide physical currency, pre-count into groups of 100, and have the groups arrange it into groups of ten for efficient payment*.

An eye on equitable action

An auction is typically fast-paced, so it is important to proceed in stages that allow opportunity for all learners to participate in the mathematical sense-making. I start by using two structures. First, the auction's 15 lots are divided into three sections of 5 lots each. Scheduling breaks between sections allows A) time for learners to gather their thoughts and B) space for the teacher to intentionally engage specific students in mathematical conversation. Second, I establish a role of "bidder" in each group. That student is responsible for holding the auction paddle and registering all bids for the group. A different student takes on the role of bidder for each section of the auction.

Remember: Equitable action does not mean that each student receives prizes at the end of the task. Rather, it means that each student is given the opportunity to participate in the mathematical sense-making!

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Task Directions

The Launch: ~ 10 minutes

- Write out the numbers 1 through 20 on a board in a prominent position in the room. This is where you will record all the dice rolls during The Payoff.
- Create student groups. Groups of 3 are large enough to generate a variety of ideas, but small enough that every student can feel heard.
- 7 – 10 groups is a nice amount for classroom activity. It's also nice to have about twice as many lots as groups. Consider adding the expansion lots if you have a large number of student groups.
- Provide each group with 100 units of currency, and organize into piles of 10.
- Give each group ***a single copy*** of the "Lots for Sale" and "Dice Distribution Charts" handouts. Having a single copy encourages collaborative activity. (Additional copies will be provided before The Auction.)
- Explain the mechanics of the auction.
    - Every lot is a possible result when rolling the two dice. After they are all purchased, the dice will be rolled 20 times.
    - Each time a lot you purchased occurs, each member of the group earns a single prize. The goal is to get as many prizes as possible.
    - The lots are auctioned in order according to their lot number.
    - A single roll might result in multiple lots collecting prizes. It is possible for your team to earn more than one prize per roll.
    - *After the auction is done, groups return any leftover currency to buy a single additional prize for each group member. (1 unit = 1 prize; 100 units = 1 prize).*
- It may help to model dice rolls for students. Examples of three groups and their purchased lots are provided on *page 9*. Each image shows a possible dice roll. In each scenario, the lots that would not collect a prize are marked with an "✕" and the lots that would collect a prize are marked with a "✓".
- Answer any clarifying questions and then have the groups *review* the list of lots for sale, *identify* the ones they would like to target, and *estimate* the maximum prices they would pay for their target lots.
- The "Lots for Sale" sheet has a *Target Lot* column for them to mark the lots they want to bid on, and an *Our Max Price* column for them to estimate their maximum prices. These will help organize group activity during auction.
- As the groups prepare for the auction, circulate among them, listening to student strategies and probing their thoughts with key questions (some of which are listed on *Page 13*).

is rolled

Group 1

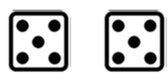
- ✗ A double is rolled
- ✗ The numbers sum to ten

Group 2

- ✓ The product of the numbers is six
- ✓ A single 3 is rolled

Group 3

- ✓ The two numbers are consecutive



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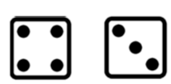


is rolled

Group 1

- ✗ A double is rolled
- ✗ The numbers sum to ten

Group 2

- ✗ The product of the numbers is six
- ✓ A single 3 is rolled

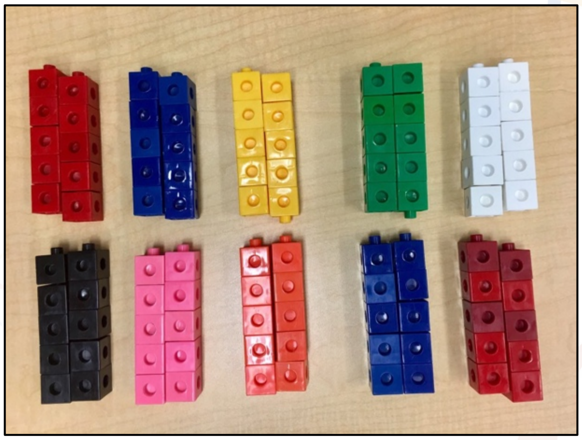
Group 3

- ✓ The two numbers are consecutive

The Auction: ~ 25 minutes

- Have each group choose a bidder. The bidder should be in clear view of the teacher. (*The role of bidder should be reassigned for each section of lots.*)
- Hand out more copies of the “Lots for Sale” handout so that each student has their own copy. Students will keep track of the auction by recording purchase price and tallying times rolled for each lot during The Auction and The Payoff.
- Open up the bidding on Lot 1. Only a group’s bidder can register bids by signalling with their paddle, but students are free to have conversations with each other to adjust their strategy while the auction is going on.
- You, as the auctioneer, can suggest the next possible bid, but students might bid lower or higher than your suggestion. That is great; it shows evidence of careful planning. For instance, the current bid might be “10”, and you could ask for “15”. A group is permitted to bid “11” or “25” as long as they shout out the value as they raise their paddle to register their bid.
- After each lot is purchased, remind each student to record the purchase price on their “Lots for Sale” handout.
- If you are using physical currency, have the group pay for their lot by nominating a student to bring up adequate currency to the front. This heightens the kinesthetic dimension of the task. If they are keeping a tally, have them subtract the purchase price from their remaining bank.
- After Lots 1-5 are purchased, allow five minutes for groups to debrief the happenings and adjust their targets and maximum prices in response to the market. During this time, circulate, listen, assist, and question the groups.
- Auction Lots 6-10 off, and then take another five minutes for adjustment and discussion. Complete the auction with Lots 11-15. This breaks The Auction into five phases:

    - Auction Lots 1-5: ~ 5 minutes
    - Group debrief / adjustment: ~ 5 minutes
    - Auction Lots 6-10: ~ 5 minutes
    - Group debrief / adjustment: ~ 5 minutes
    - Auction Lots 11-15: ~ 5 minutes



The Payoff: ~ 15 minutes

- Roll the two dice. After each roll, write the result on the board space you previously prepared.
- Have students raise their hand if they feel they should collect a prize from the roll. (*It is easiest if prizes are not handed out until the 20 rolls are complete.*)
- Call on the groups and have them respond with their successful lot(s). (*It is helpful if they refer to them by lot number as well as lot description.*)
- Remind students to place tally marks on their "Lots for Sale" handout next to each lot that collects a prize.
- Repeat this rolling process all twenty times. (During this process, shouts of celebration and disgust might initiate interesting discussions about expectations, likelihood, and chance.)
- After the twenty rolls, give each student a copy of the "Dice Rolls" handout and have them copy down the rolls from the board. Having a copy of the rolls will help students answer the reflection questions.
- While students fill out the "Dice Rolls" handout, you can walk around the room and deliver the prizes to the groups.

The Reflection: ~ 10 minutes

- After the auction, give each student a copy of the “Personal Reflection” handout. The four questions posed are excellent topics to guide students’ reflection. If students have the handout in front of them during a discussion, they can make notes to themselves to help with completing the sheet at a later time.
    - I do not declare “winners” and “losers” in each round, because I think this does little to encourage further activity. Rather, the reflection is focused on the lots. You can ask the group: Did the dice roll as we expected? Did any rare events occur?
    - Students tend to want to make grandiose statements about luck (e.g. “That should have *never* happened”). These are teachable moments. Take a moment to compute the probabilities associated with their claims, and pair the mathematics to their intuition and enthusiasm.
    - Students also tend to fixate on clusters of similar events (e.g. four consecutive rolls contained a 1). This a great moment to transition their thinking from using the dice chart to calculate the probability of a single event and toward how probabilities of independent events interact.
    - I like to have students complete the bulk of the reflection questions outside of class time, which provides ample time to think about the connections between what happened (experimental probability) and what should have happened (theoretical probability). It also provides space for those students who are less comfortable in discussion-based activities.
- Before they leave, each student should have a copy of a completed “Lots for Sale” and “Dice Rolls” handout; this information will become a major part of their justifications to the reflection questions.
- One interesting possibility, if time allows, is to repeat the auction after a detailed reflection in order to observe how students change their actions after having more time to work with the probabilities.

Anticipated Student Action

Use dice charts to count the frequencies

Students might use the dice distributions to count how many ways a lot can occur. This is known as a lot's frequency. For instance, *Lot 3: Both numbers are greater than or equal to 5* has a frequency of 4.

The likelihood of events can be compared using these frequencies, but students should also be introduced to the idea of probabilities. A probability is the frequency of favorable outcomes divided by the total number of possible events, which results in a value between 0 and 1.

Confuse the frequency of favorable outcomes with the number of expected prizes

A student may assume that the frequency of a lot in the dice distribution chart is how many prizes they will receive, but the dice will only be rolled 20 times, not 36.

This student needs to understand that a probability predicts the likelihood of success on *any* roll, and multiplying the probability by the number of rolls will result in the expected number of prizes.

Compare the probabilities between lots to determine good deals

Students might rank the lots in order of likelihood using frequencies or probabilities. Encourage these students to compare likelihood using probabilities. They can do this in either fraction or decimal form.

Once they determine the likelihood of lots, they can compare to similar lots auctioned previously to predict a reasonable maximum price. The comparison between probability and price also provides an argument for which lots were the best deals.

Try to purchase several less-probable lots

Students might assume that groups will spend top dollar for very likely events. This might leave an opportunity to purchase several less-likely events at bargain prices. The best lots in Dice Auction are not always those with the highest probabilities; deals are determined by comparing probabilities to prices.

Spread out their purchased lots across dice distribution

Students might target lots that collect on different dice rolls rather than lots that all collect on identical rolls. For instance, *Lot 4: The product of the numbers is 6* and *Lot 8: The two numbers are consecutive* both collect when a 2&3 or 3&2 are rolled, while *Lot 1: A double is rolled* and *Lot 14: A single 1 is rolled* never collect on the same dice roll. This increases the odds that they collect on any given roll, but decreases the probability that they collect multiple prizes on any one roll.

Key Questions

What is the probability of this lot occurring?

- Use this question to have students show evidence that they can construct the probability of an event by dividing the frequency of favorable outcomes by the total number of outcomes.

Which lot is more likely and why?

- Use this question to encourage students to calculate probabilities and compare the values.

Is there a similar lot that has a slightly higher or slightly lower probability?

- Use this question to make connections between lots when students are trying to determine a fair auction price.

Which of these two are the best deal?

- Use this question to talk about the ratio between a lot's purchase price and probability of the lot being rolled.

How many times do you expect for this lot to occur in the twenty rolls?

- Use this question to encourage students to make the distinction between the frequency and the probability of a specific lot.

Extensions and Adaptations

Expansion list of auction lots

I have provided an expansion list of auction lots that you can use if your class is large. You can expand the Dice Auction by including both the original and expansion lists of lots. It will take more time, but it is nice to have enough lots so that each group can make purchases. The list also includes a few blank rows so that students can create their own lots.

You can plan to use these from the onset, or surprise your class with extra lots after the bidding for the first fifteen are done. I have used the surprise method when there was a lot of currency left over after the original auction was complete.

The lots are provided on the handout “Expansion Auction Lots” available in the “Materials for Copying” section of this lesson plan.

A new pair of dice

In this extension, you have students consider how the probability of a lot is affected if the type of dice were switched. For instance, would “a single 1 is rolled” become more or less likely if you rolled a pair of 10-sided dice? How would “the numbers sum to 10” be affected? What about “the product of the numbers is 6”? Are there any lots that would be unaffected by this change in dice?

Appeal to the gambler’s fallacy

The gambler’s fallacy is the belief that if a specific lot has occurred more than expected, then its likelihood of occurring in the future drops in order to balance this out. This effect works the other way as well: If a lot has not occurred, it becomes more likely because it is “due” to happen. After the 20 rolls, some lots will have inevitably underperformed or overperformed. Pick an obvious one, and ask the class if the probability of it occurring has changed. I enjoy playing a “double devil’s advocate” role in this discussion. This is where I agree that both sides seem to make sense. If a lot has underperformed, I argue that it must occur soon because it’s “due.” I then also suggest that it won’t happen because the dice are “cold.” In actuality, neither effect is true. The rolls are independent and therefore previous rolls have no effect on the theoretical probability—a critical understanding in the probability of compound events.

Theoretical Probability Calculations

These calculations are for teacher reference and convenience only. Students should calculate the theoretical probabilities as a part of their auction activity.

These calculations play a critical role in determining the market value of lots, but remember that other factors will affect the market such as the number of lots remaining, precedent set on previous lots, amount of currency groups have to spend, appetite for risk, knowledge of classmates, etc.

Outcomes listed D1&D2: D1 = Result of First Die D2 = Result of Second Die

Lots for Sale		
Lot Number	Favorable Outcomes	Theoretical Probability
1	1&1, 2&2, 3&3, 4&4, 5&5, 6&6.	$\frac{6\ doubles}{36\ total} = \frac{1}{6} \approx 16.67\%$
2	Four apart: 1&5, 5&1, 2&6, 6&2. Five apart: 1&6, 6&1.	$\frac{6\ favorable}{36\ total} = \frac{1}{6} \approx 16.67\%$
3	5&5, 5&6, 6&5.	$\frac{3\ favorable}{36\ total} = \frac{1}{12} \approx 8.33\%$
4	1&6, 6&1, 2&3, 3&2.	$\frac{4\ favorable}{36\ total} = \frac{1}{9} \approx 11.11\%$
5	3&4, 4&3.	$\frac{2\ favorable}{36\ total} = \frac{1}{18} \approx 5.56\%$
6	1&3, 3&1, 2&2, 3&6, 6&3, 4&5, 5&4.	$\frac{7\ favorable}{36\ total} \approx 19.44\%$
7	Case 1: Roll a double and match that double Case 2: Roll a non-double and match that non-double	Case 1: $\frac{6}{36} \cdot \frac{1}{36} = \frac{6}{1296}$ Case 2: $\frac{30}{36} \cdot \frac{2}{36} = \frac{60}{1296}$ Total: $\frac{6+60}{1296} = \frac{66}{1296} = \frac{11}{216} \approx 5.09\%$
8	1&2, 2&1, 2&3, 3&2, 3&4, 4&3, 4&5, 5&4, 5&6, 6&5.	$\frac{10\ favorable}{36\ total} = \frac{5}{18} \approx 27.78\%$
9	1&3, 3&1, 2&3, 3&2, 4&3, 3&4, 5&3, 3&5, 6&3, 3&6.	$\frac{10\ favorable}{36\ total} \approx 27.78\%$
10	1&1.	$\frac{1\ favorable}{36\ total} \approx 2.78\%$
11	Assume all lots (in original 15) have been purchased. The following rolls result in no lots collecting prizes: 2&4, 4&2, 2&5, 5&2.	$\frac{4\ favorable}{36\ total} = \frac{1}{9} \approx 11.11\%$

12	1&2, 2&1, 1&3, 3&1, 1&5, 5&1.	$\frac{6\ favorable}{36\ total} = \frac{1}{6} \approx 16.67\%$
13	2&6, 6&2, 3&4, 4&3.	$\frac{4\ favorable}{36\ total} = \frac{1}{9} \approx 11.11\%$
14	1&2, 2&1, 1&3, 3&1, 1&4, 4&1, 1&5, 5&1, 1&6, 6&1.	$\frac{10\ favorable}{36\ total} = \frac{5}{18} \approx 27.78\%$
15	4&6, 6&4, 5&5.	$\frac{3\ favorable}{36\ total} = \frac{1}{12} \approx 8.33\%$

Expansion Auction Lots		
Lot Number	Favorable Outcomes	Theoretical Probability
16	1&3, 3&1, 2&2.	$\frac{3\ favorable}{36\ total} = \frac{1}{12} \approx 8.33\%$
17	2&5, 5&2, 4&5, 5&4, 5&6, 6&5.	$\frac{6\ favorable}{36\ total} = \frac{1}{6} \approx 16.67\%$
18	1&1, 1&3, 3&1, 1&5, 5&1, 3&3, 3&5, 5&3, 5&5.	$\frac{9\ favorable}{36\ total} = \frac{1}{4} \approx 25.00\%$
19	This probability depends on the other lots purchased by the owner of this lot.	
20	1&3, 3&1, 2&3, 3&2, 3&3.	$\frac{5\ favorable}{36\ total} \approx 13.89\%$
21	1&5, 5&1, 2&6, 6&2.	$\frac{4\ favorable}{36\ total} = \frac{1}{9} \approx 11.11\%$
22	4&6, 6&4, 5&5, 5&6, 6&5, 6&6.	$\frac{6\ favorable}{36\ total} = \frac{1}{6} \approx 16.67\%$
23	Sum of 2 twice: $\frac{1}{36}\cdot\frac{1}{36}$ Sum of 3 twice: $\frac{2}{36}\cdot\frac{2}{36}$ Sum of 4 twice: $\frac{3}{36}\cdot\frac{3}{36}$ Sum of 5 twice: $\frac{4}{36}\cdot\frac{4}{36}$ Sum of 6 twice: $\frac{5}{36}\cdot\frac{5}{36}$ Sum of 7 twice: $\frac{6}{36}\cdot\frac{6}{36}$ Sum of 8 twice: $\frac{5}{36}\cdot\frac{5}{36}$ Sum of 9 twice: $\frac{4}{36}\cdot\frac{4}{36}$ Sum of 10 twice: $\frac{3}{36}\cdot\frac{3}{36}$ Sum of 11 twice: $\frac{2}{36}\cdot\frac{2}{36}$ Sum of 12 twice: $\frac{1}{36}\cdot\frac{1}{36}$ Total: $\frac{1+4+9+16+25+36+25+16+9+4+1}{36\cdot 36} = \frac{146}{1296} = \frac{73}{648} \approx 11.27\%$	
24	2&6, 6&2, 3&5, 5&3, 4&4.	$\frac{5\ favorable}{36\ total} \approx 13.89\%$
25	1&1, 1&2, 2&1, 1&3, 3&1, 1&4, 4&1, 1&5, 5&1, 1&6, 6&1.	$\frac{11\ favorable}{36\ total} \approx 30.56\%$

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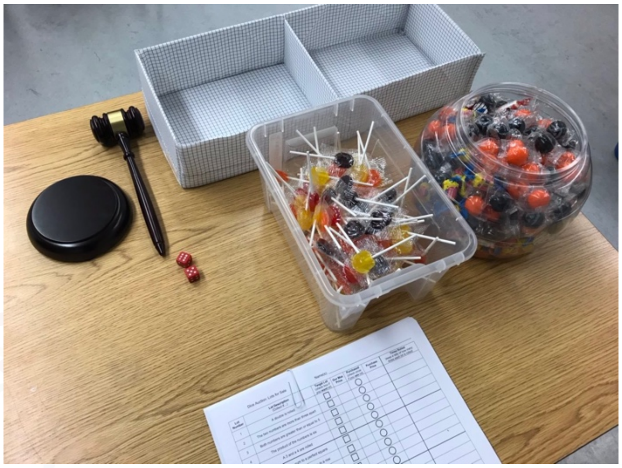
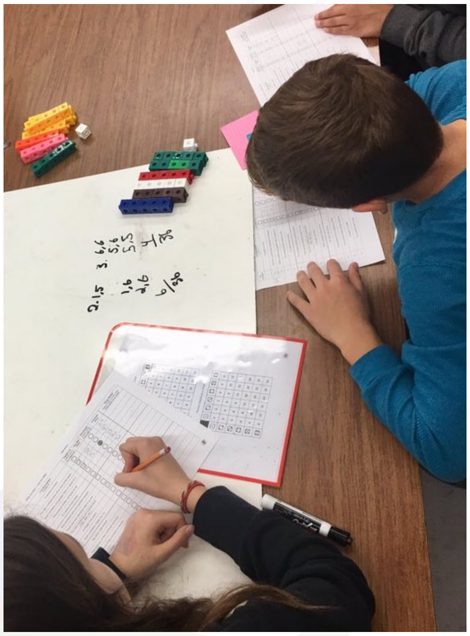
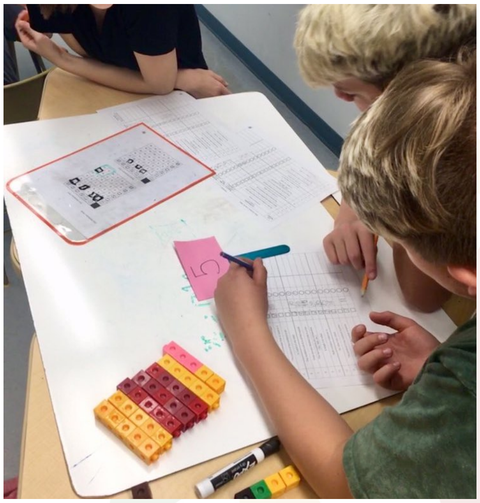
Samples of Student Work



The teacher's auction station set up before the auction begins.



Student groups use a collaborative workspace, dice distribution charts, and their understanding of probability to prepare for auction.



Student responses to the reflection questions:

Responses like these, taken directly from student work, can be used to orchestrate a conversation around the events of the Dice Auction.

Q1: Explain your strategy for the Dice Auction. What types of lots were you targeting and why?

"We went for the least likely ones because they would go for the least money."

"We decided to go big or go home for number 8. Because it would have the best rolling results, and we were right."

"It was mostly just finding the probability and waiting out which ones caught on."

Q2: Which lot had the most unexpected ***bidding*** results? Why do you think that was?

"Lot 6 had a 7/36 probability which means it's pretty likely to happen...They spent $46 on lot 6 which was a really high price."

"Lot 10 sold for $18 but it had half the chances of lot 4 that sold for $20."

"We wanted lot 10 because it was unlikely and thought we could buy it cheap, but the bidding went higher than we wanted."

"The two numbers are more than three apart [lot 2] went for $40 right after $29 for a double is rolled [lot 1]. They have the same probability."

Q3: Which lot had the most unexpected ***rolling*** results? Why do you think that was?

"Lot 8 rolled 8 times. Lot 8 was where the two numbers were consecutive. $4/36 \approx 2/20$. It was really unexpected because it should only happen twice."

"A single one is rolled because it was rolled 6 times in a row."

"The one that no one was really interested in was lot 12 that my group got for $20. It had 1/6 probability but ended up winning 5 times."

"I thought lot 4 would get rolled at least once but it didn't. Even though the probability was 4/36 which is about 2/20. The exact thing happened with lot 5 and it had the same probability."

Q4: Based on the prices paid at the Dice Auction, how much would you expect to pay for the following lot at auction: *Collect if one of the numbers is a perfect square*.

"Since lot 6 (the numbers sum to perfect square) sold for a whopping $46, I believe that this one would sell for even more. The probability of it happening is 12/36."

"The chances of rolling a 1 or a 4 on a 6 faced dice is 2/6. I think the range for this lot should be $25-$35."

"Well if you roll a 1 or a 4, you get it. So that's 12 times of 36. I would spend a lot."

"Tricky because you can't get a 1 and a 4. But 8 ways is still pretty high."

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Materials for Copying

The following five pages contain the task handouts. These five handouts are ready to copy and use with your students. Each is labeled at the top.

This section includes:

1. "Lots for Sale"

   Additional clarification of specific lots:

   - *Lot 7: There are two identical rolls in a row*. If the numbers on the dice match exactly the previous roll, the owner of this lot receives a prize. The order of the dice does not matter. (i.e. rolling a 4&5 after rolling a 5&4 counts).
   - *Lot 11: No other lots collect*. If the dice are rolled and the result does not award any prizes to the class, the owner of this lot receives a prize.

2. "Dice Distribution Charts"

3. "Dice Rolls"

4. "Personal Reflection"

5. "Expansion Auction Lots"

   Additional clarification of specific lots:

   - *Lot 19: You have not collected on three rolls in a row*. If the dice are rolled and it is the third roll in a row that the owner of this lot has not collected a prize, they collect a prize. Winning a prize on this lot counts as winning a prize and their missed rolls resets to zero.

Dice Auction: Lots for Sale Name(s): ______________________________

Lot Number	**Lot Description** (Collect if…)	**Target Lot** (*check box if you **want** lot*)	**Our Max Price**	**Purchased** (*check circle if you **win** lot*)	**Purchase Price**	**Times Rolled** (*keep a tally of how many times each lot is rolled*)
1	A double is rolled	☐		○		
2	The two numbers are more than three apart	☐		○		
3	Both numbers are greater than or equal to 5	☐		○		
4	The product of the numbers is six	☐		○		
5	A 3 and a 4 are rolled	☐		○		
6	The numbers sum to a perfect square	☐		○		
7	There are two identical rolls in a row	☐		○		
8	The two numbers are consecutive (1&2, 4&5, etc.)	☐		○		
9	A single 3 is rolled	☐		○		
10	Both numbers rolled are 1s	☐		○		
11	No other lots collect	☐		○		
12	The product of the numbers is prime	☐		○		
13	The product of the number is twelve	☐		○		
14	A single 1 is rolled	☐		○		
15	The numbers sum to ten	☐		○		

Dice Distribution Charts

Sum Distribution Chart

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

Product Distribution Chart

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	1	2	3	4	5	6
⚁	2	4	6	8	10	12
⚂	3	6	9	12	15	18
⚃	4	8	12	16	20	24
⚄	5	10	15	20	25	30
⚅	6	12	18	24	30	36

Dice Auction: Dice Rolls Name: ______________________________

Record the results of each roll from the dice auction in the table below.

These results will be important pieces of information to answer the reflection questions.

1.	2.	3.	4.
5.	6.	7.	8.
9.	10.	11.	12.
13.	14.	15.	16.
17.	18.	19.	20.

Dice Auction: Personal Reflection Name: ______________________________

Address each of the following questions with a mathematical argument. Explain your argument in words, but feel free to use mathematical symbols or diagrams to support your reasoning.

1) Explain your strategy for the Dice Auction. What types of lots were you targeting and why?

2) Which lot had the most unexpected ***bidding*** results?

(*i.e. More or fewer groups were interested in the lot than you predicted*).

Why do you think that was?

3) Which lot had the most unexpected ***rolling*** results?

(*i.e. The lot collected more or fewer times than you predicted*).

Why do you think that was?

4) Based on the prices paid at the Dice Auction, how much would you expect to pay for the following lot at auction:

"*Collect if one of the numbers is a perfect square*"

Justify your argument.

Dice Auction: Expansion Auction Lots Name(s): ______________________________

Lot Number	Lot Description (Collect if…)	Target Lot (*check box if you **want** lot*)	Our Max Price	Purchased (*check circle if you **win** lot*)	Purchase Price	Times Rolled (*keep a tally of how many times each lot is rolled*)
16	The numbers sum to four	☐		○		
17	The product of the numbers is a multiple of 10	☐		○		
18	The product of the numbers is odd	☐		○		
19	You have not collected on three rolls in a row	☐		○		
20	A 3 is the highest number rolled	☐		○		
21	The numbers have a difference of four	☐		○		
22	The sum of the numbers is double digits	☐		○		
23	Two rolls in a row have an identical sum	☐		○		
24	The numbers sum to eight	☐		○		
25	The sum of the numbers is greater than the product of the numbers	☐		○		
		☐		○		
		☐		○		
		☐		○		
		☐		○		
		☐		○		