How to Solve the Rubik's Cube: All 6 Sides!

Daniel Goodman and Daniel Rose-Levine

Introductions

Daniel Goodman

- 18 years old, freshman at Princeton University
- From Howell, NJ
- Cubing for 7 years
- 2018 Pyraminx US national champion
- Currently ranked 7th in the world with a 2.32 second average
- YouTube channel with 50,000 subscribers - DGCubes

Daniel Rose-Levine

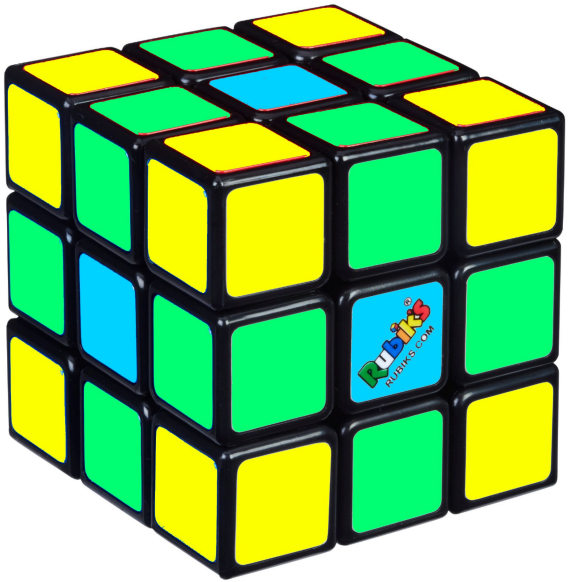
- 16 years old, freshman at Bard College at Simon’s Rock
- From Red Hook, NY
- Cubing for 4 years
- 3x3 with feet world record single (16.96) and average (22.22)
- 1st in North America for sum of ranks

Demonstration!

The Basics

Types of pieces

- A Rubik’s Cube is made up of edges, corners, and centers
- Edges have 2 stickers, corners have 3, and centers have 1



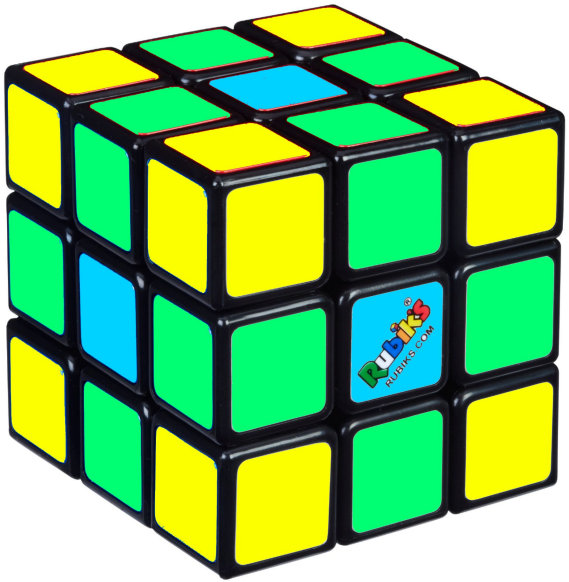
Edges in green

Corners in yellow

Centers in blue

Types of pieces

- Centers don’t move in relation to each other!
- That means, the center determines what color the side will be when solved
- That also means that opposite centers will stay the same!



Edges in green

Corners in yellow

Centers in blue

On a standard cube:

Blue is opposite green

White is opposite yellow

Red is opposite orange

When the pieces are solved

- An edge piece is solved if both stickers match both centers
- A corner piece is solved if all three stickers match all three centers
- All pieces have ONE place they belong when solved

Is this side solved?

No! In fact, none of those orange pieces appear solved!

What does a solved side look like?

Misconception!

- You don’t solve the first side, then the second, then the third, then the fourth, then the fifth, then the sixth
- Once you have one solved side, you’ve already made quite a bit of progress on quite a few other sides!
- Focusing on only one of those sides at a time would be extremely difficult and not particularly fast

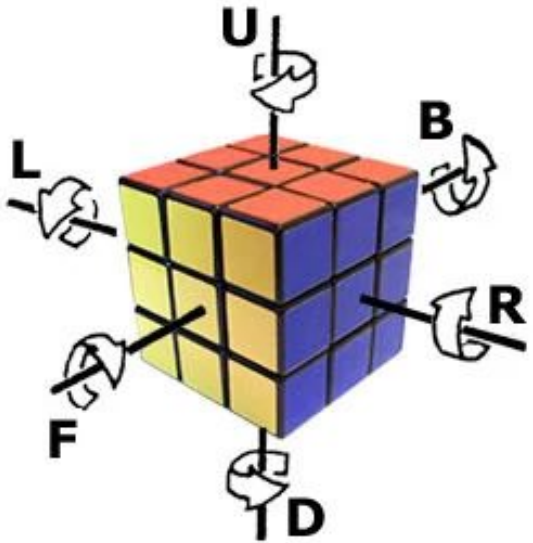
Misconception!

- Instead, we'll be solving layer-by-layer!
- Notice how when you have a solved side, an entire THIRD of the cube is done
- We'll take advantage of this

Notation

Notation?

- A way of describing the moves you make on a cube
- There are six sides, so we'll represent each side with a letter



Up, **D**own,

Left, **R**ight,

Front, **B**ack

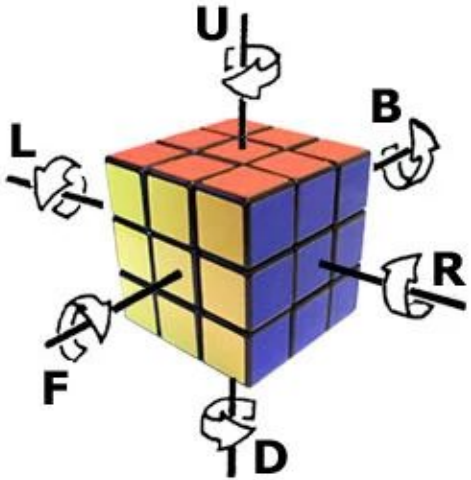
Normally, we'll hold the cube so we can see the U, F, and R sides

Notation!

- You can make three different turns on each side
  - One turn clockwise
  - One turn counterclockwise (same as three turns clockwise) - add a ’
  - Two turns (same in both directions) - add a 2

“Clockwise” is based on if the side was facing us

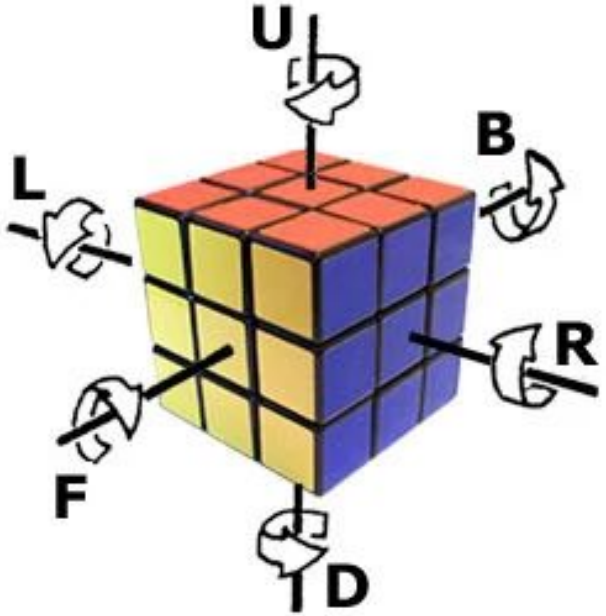
Turning F clockwise goes in the same direction as turning B counterclockwise (this is true for all opposite sides!)



Examples

- Turn the front face twice - F2
- Turn the right face counterclockwise once - R’
- Turn the bottom face clockwise once - D
- Turn the left face twice - L2

Notice: the letter you use is based on the way you’re holding the cube, not the color of the side you’re turning!



Try it out!

- We can put these moves together to create **algorithms** - a fancy word for sequences of moves
- Try it out by doing these algorithms!
- R U R’ U’
- F2 B2 R2 L2 U2 D2
- R’ F R F’

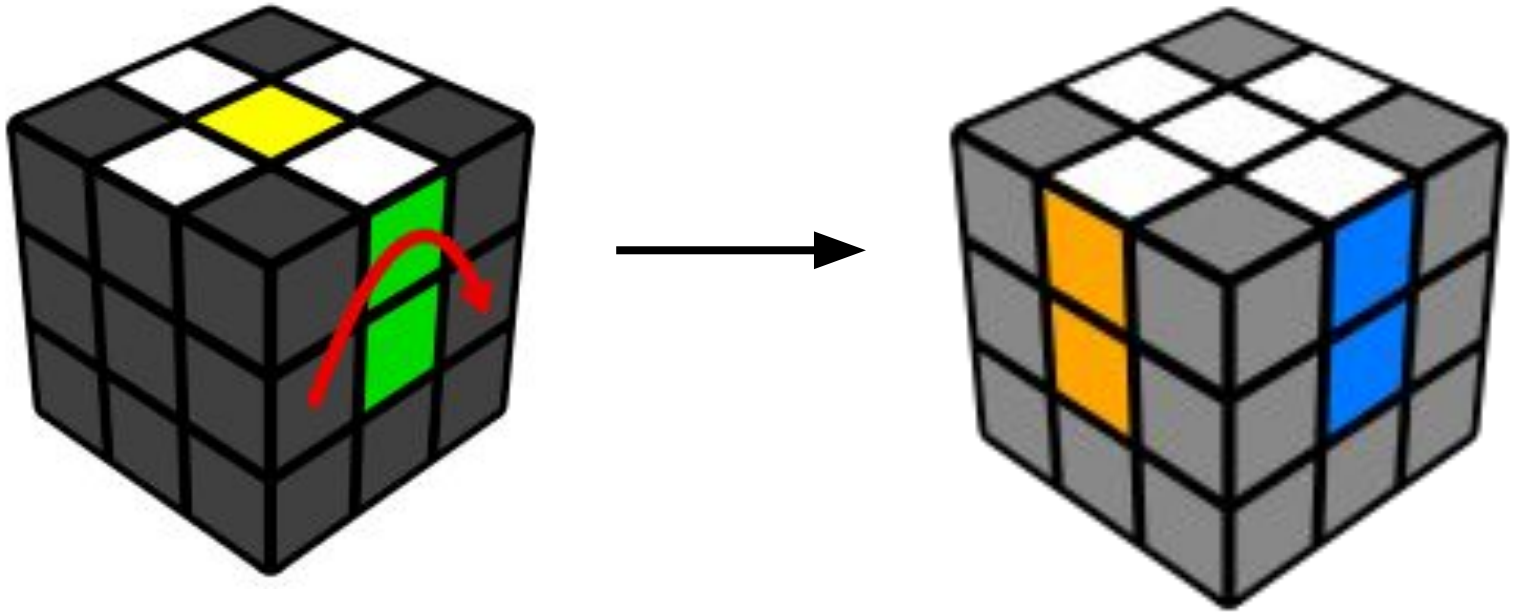
The first layer

Daisy

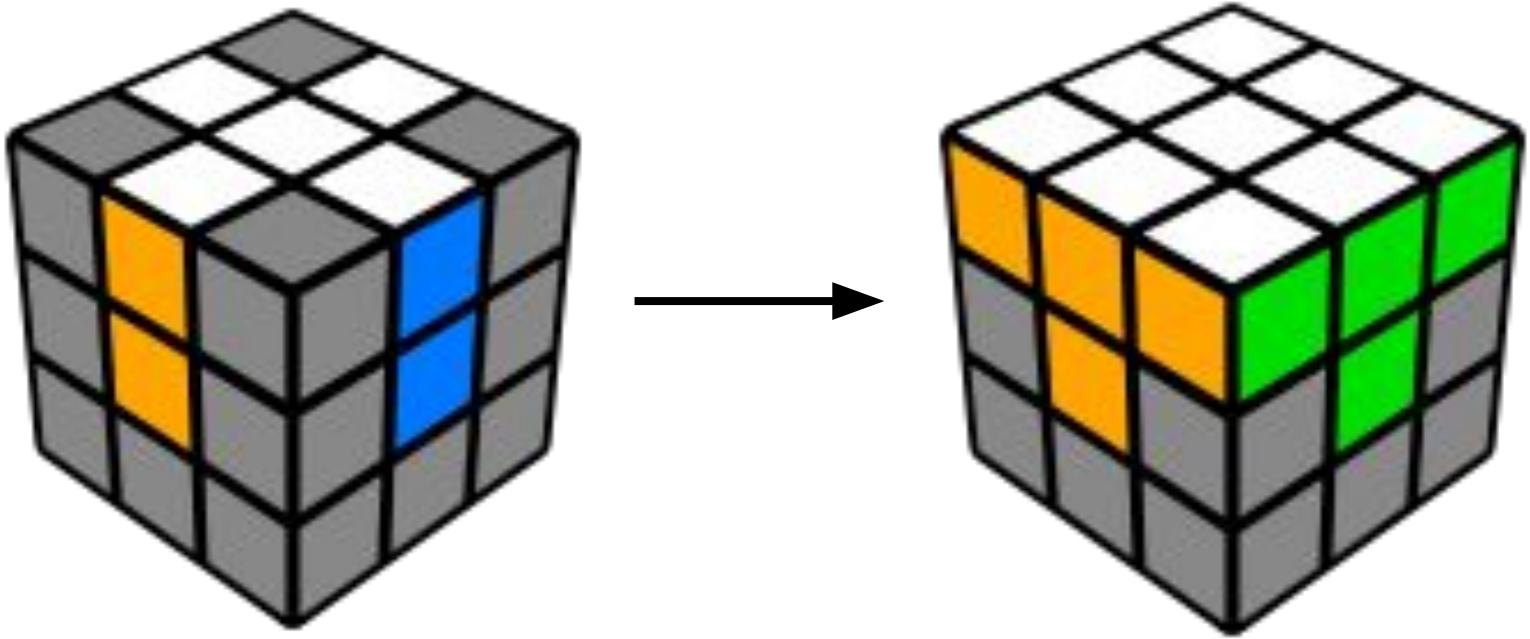
First, get all 4 white edges around the yellow center. This step is completely intuitive and can be a bit tricky at first, but keep trying!

White cross

Using **U moves**, match a white edge with its corresponding center. Then, do an **F2** to move the edge to the bottom. Repeat for all 4 white edges.

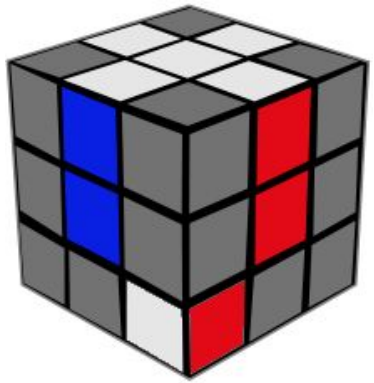


Corners



Corners

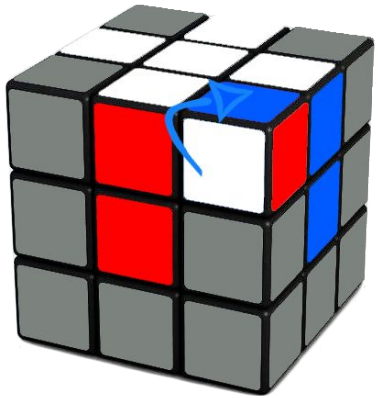
Find a white corner in the bottom layer. Using **D moves**, position it on the bottom right, directly under where it must go to be solved. Repeat the algorithm until the corner is solved.



R’ D’ R D

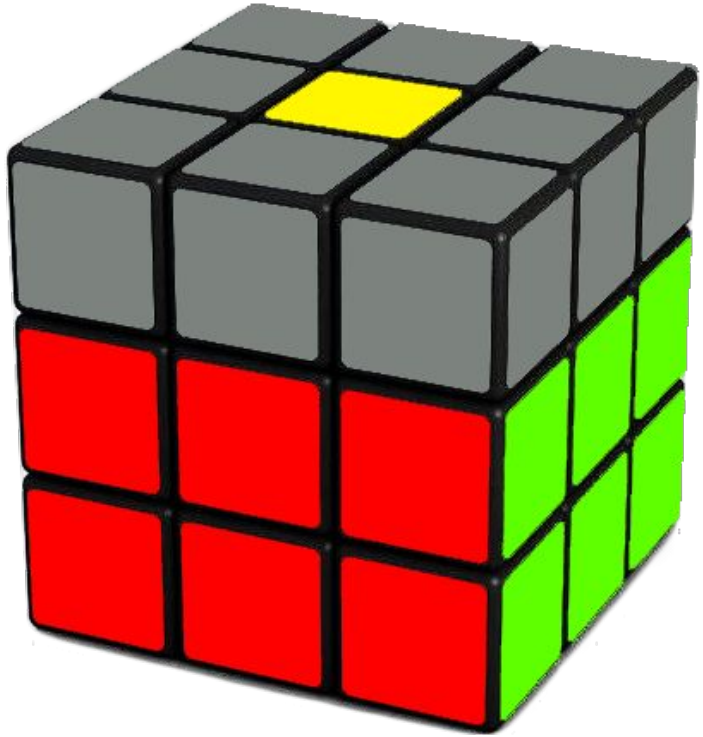
Corners

If there are no white corners in the bottom layer, position the top layer such that an unsolved corner is in the front-right position. Then do the algorithm once to bring this corner into the bottom layer.



R’ D’ R D

The second layer



Second layer

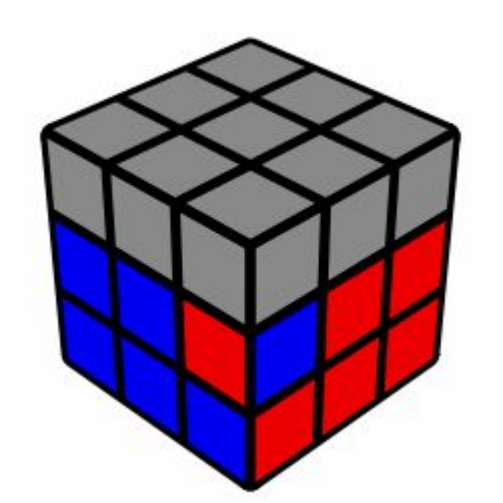
Find a second layer edge (an edge without any yellow on it) in the top layer and using **U moves** match it up with the corresponding center

U R U' R' U' F' U F

U' L' U L U F U' F'

Second layer

Sometimes you won’t have any second layer edges left in the top layer. In this case, find an unsolved edge in the second layer, and perform either algorithm.



U R U’ R’ U’ F’ U F

OR

U’ L’ U L U F U’ F’

Remember, you must position the cube differently depending on which algorithm you choose!

A quick math break

A quick math break

How many ways can you scramble a Rubik’s Cube?

Edges

- 12 edge pieces = 12 * 11 * 10 * 9… = 12! ways these pieces can be arranged
- Each one can be flipped/oriented 2 ways = 2 * 2 * 2 * 2… = 2^{12} ways these pieces can be flipped once their location is chosen
- Since these two things are independent of each other, we multiply them

$$12! * 2^{12}$$

Corners

- 8 corner pieces = 8 * 7 * 6 * 5… = 8! ways these pieces can be arranged
- Each one can be flipped/oriented 3 ways = 3 * 3 * 3 * 3… = 3^8 ways these pieces can be flipped once their location is chosen
- Since these two things are independent of each other, we multiply them

$$8! * 3^8$$

Putting it all together

- Corners and edges are independent of each other, so we multiply the two numbers we already got

$$(12! * 2^{12}) * (8! * 3^{8})$$

Putting it all together

- This might look like the final answer, but there's actually one secret we left out:
- Certain permutations are not possible!
  - You can't have just one edge flipped (or any odd number) because no turn affects the orientation of an even number of edges
  - So, we divide by two because half of our permutations have an odd # flipped

$$\frac{(12! * 2^{12}) * (8! * 3^{8})}{2 * 3 * 2}$$

Final answer!

Accounting for all of this, we get our final answer…

$$43{,}252{,}003{,}274{,}489{,}856{,}000$$

$$\approx$$

43 quintillion permutations

Fun fact

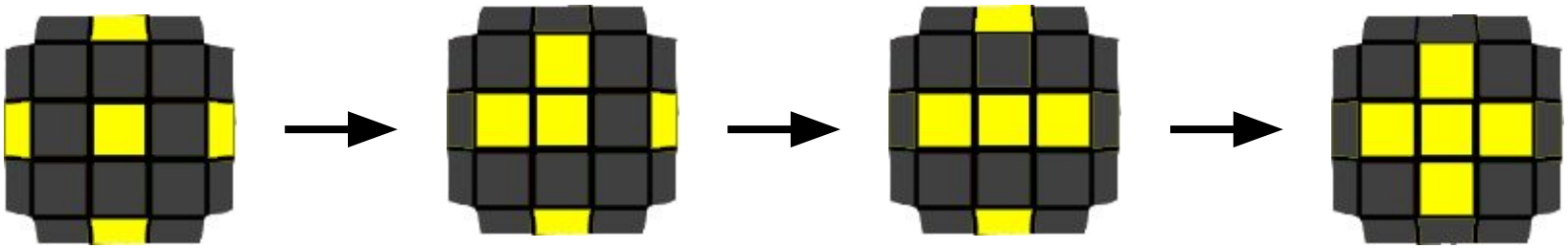
- The fastest cubers in the world can reach solves around 10 turns per second
- If you turned at 10 turns per second and never went through the same permutation twice, it would take you **137,057,327,789 years** to see all the permutations of the cube!
- In fact, if you got everyone else in the world to turn their cubes at 10 TPS, it’d still take **almost 18 years non-stop**!

Back to cubing

The last layer

Edge orientation

F R U R’ U’ F’



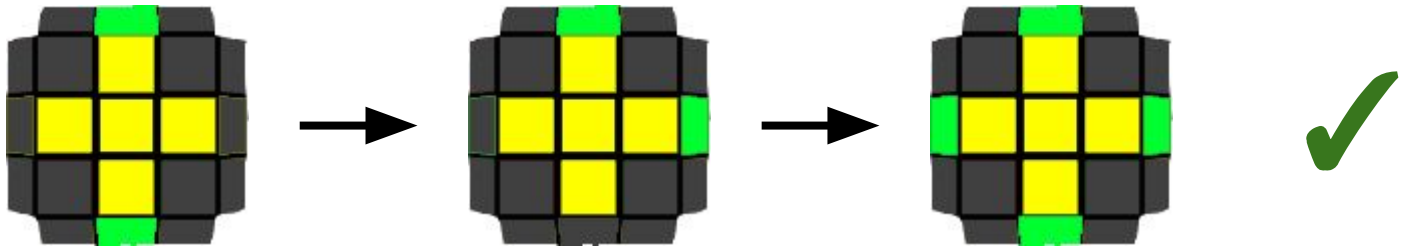
Note: it doesn’t matter if the edges are actually solved yet! All that matters is that we orient them the right way.

✓

Edge permutation

First, do U moves until you have **2** or **4** edges **solved** (actually in the right place)
Green represents a SOLVED edge, the actual color doesn't matter

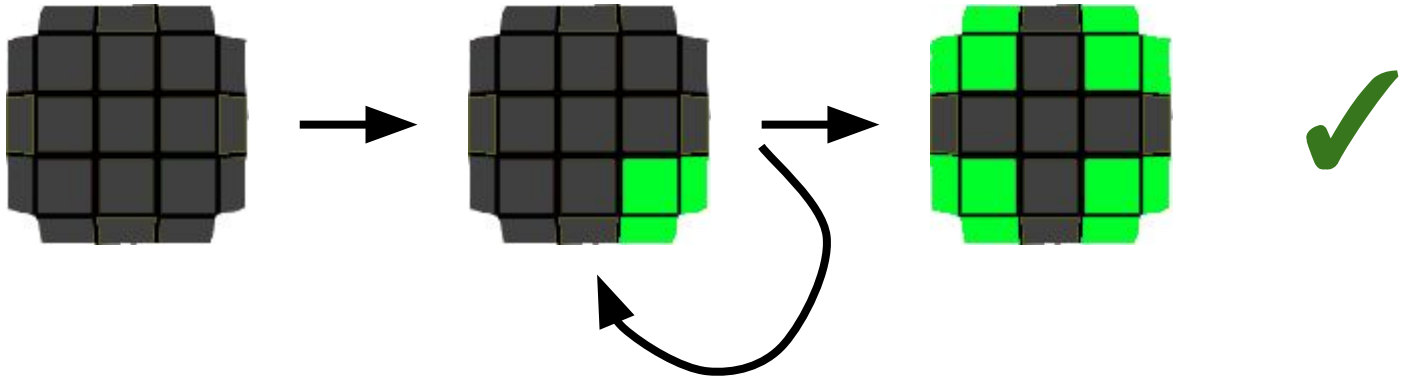
R U R' U R U2 R'



Corner permutation

Green represents a corner in the right place (not necessarily twisted right!), the actual color doesn’t matter

U R U’ L’ U R’ U’ L



Corner orientation

1. Put any un-oriented corner in the front-right position
2. Repeat R’ D’ R D (either 2 or 4 times) until the corner is solved - you will mess up the rest of the cube in this process, but it will be restored if you follow the instructions carefully!
3. Put any other un-oriented corner in the front-right position by turning the U face (do NOT rotate the cube, this will mess up the rest!)
4. Go back to step 2, and continue this process until all the corners are solved

Not solved? There are a couple possibilities…

Congrats!!

But now what?

- After you've solved the cube once, there are still a lot of fun things to do!
- Memorize the algorithms! (We've put them all on the last slide so you can take a look when you go home)
- Practice!
- Try new puzzles - 2x2, 4x4, 5x5, Pyraminx, Megaminx, etc.
- Try new methods - CFOP, Roux, ZZ, etc.
- Learn more cube theory - commutators!
- Try solving one-handed, with feet, etc.

Algorithms

First layer

Daisy - no algorithms

Cross - no algorithms

Corners - R’ D’ R D

Second layer

Right - U R U’ R’ U’ F’ U F

Left - U’ L’ U L U F U’ F’

Last layer

Edge orientation - F R U R’ U’ F’

Edge permutation - R U R’ U R U2 R’

Corner permutation - U R U’ L’ U R’ U’ L

Corner orientation - R’ D’ R D

Questions?