

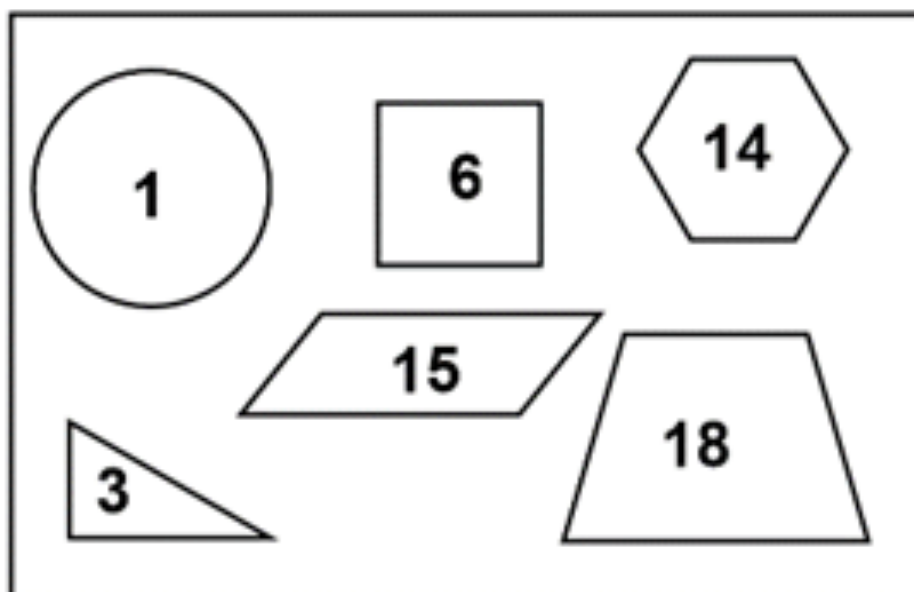
THE 2018 ROSENTHAL PRIZE

for Innovation in Math Teaching

Geometry Project: DARTBOARD

Geometric Probability
Theoretical Probability and Experimental
Probability

Elizabeth Masslich



Geometry grades 6-12

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Lesson Goals and Standards

Lesson Goals:

- Measure lengths of line segments with precision to the nearest tenth.
- Calculate areas using formulas: triangle, circle, rectangle, rhombus, regular polygon, kite, trapezoid.
- Determine geometric probabilities with targets that a random point hits a particular shape.
 - Calculate the theoretical probability.
 - Calculate the experimental probability.
- Know that the probability of a dart hitting any of the shapes is the same as the ratio of each area to the total dartboard area.
- Create a Google Spreadsheet that calculates formulas and is organized. Show your understanding of theoretical probability and of experimental probability by typing responses on a sheet.

Mathematics Standards for Mathematical Practice

- Construct viable arguments.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.

Mathematics Standards for Mathematical Content

- Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities. 6.RP.3.d
- Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems. 6.G.1
- Know the formulas for the area of a circle and use it to solve problems. 7.G.4.a
- Investigate chance processes and develop, use, and evaluate probability models. 7.SP.5 and 7.SP.6
- Understand and evaluate random processes underlying statistical experiments. S-IC.1 and S-IC.2

Mathematics ACT College and Career Readiness Standards

- Understand the role of randomization in surveys, experiments, and observational studies. S 703
- Recognize that part of the power of statistical modeling comes from looking at regularity in the differences between actual values and model values. S 705
- Recognized that when a statistical model is used, model values typically differ from actual values. S 506
- Recognize that when data summaries are reported in the real world, results are often rounded and must be interpreted as having appropriate precision. S 505
- Use the relationship between the probability of an event and the probability of its complement. S 305
- Compute straightforward probabilities for common situations. S 503
- Determine the probability of a simple event. S 403
- Perform common conversions of length within a measurement system. G203
- Compute the areas of rectangles, triangles, circles, and composite geometric figures. G303, G403, G702
- Recognize that real-world measurements are typically imprecise and that an appropriate level of precision is related to the measuring device and procedure. G 504

Preview to the Project: Active Learning in the Classroom

Materials:

- 10 inches by 12 inches of a sheet of paper
- 5 inches by 6 inches of a sheet of paper
- 10 markers, all different colors

Directions:

- The teacher or the students can do the following:
 - Create the dartboard: Cut one rectangular piece of paper that is 10 inches by 12 inches (easiest to do this from using an 11 inch by 17 inch sheet of paper).
 - Create the target: Cut one rectangular piece of paper that is 5 inches by 6 inches.
 - Tape or glue the target to the dartboard. Frame the dartboard with paper around it so that random marker hits will mark up the frame and not the wall. Then, place the framed dartboard on a wall or on a vertical whiteboard.
- Split the class into 10 groups. Each group is assigned one of the marker colors.
- Each of the 10 groups will discuss their answers to the following questions:
 - Out of 100 random marker hits, how many should land in the target? Why? Justify your answer mathematically.
 - What is the area of the target?
 - What is the area of the dartboard?
 - What is the ratio of the area of the target to the area of the dartboard?
 - Would you get different results for any of these questions if you moved the target?
- While the groups are answering the questions, have each group walk up to the dartboard at different times. They must use their marker color and randomly place “darts” on the dartboard with their marker.
 - Students should face away from the dartboard, turn around, and randomly mark the dartboard with the marker.
 - Each group should do this 10 times, so each marker has 10 hits (10 darts per marker color).
 - Students should record their “hits” and their “misses” (you can call these “in” and “out” of the target).
 - Turning around can be fun, but also can create dizzy students, so the teacher will need to monitor this active learning activity so that students don’t hurt themselves. Perhaps, limit one turn at a time so that students are not constantly turning around to make marks.
- Once all groups have placed their marker “darts” on the dartboard, total up the table values.
 - Discuss the answers to the four questions.
 - Discuss how accurate your class was.
 - Discuss how your class could be more accurate with their “darts”.
 - Discuss how your class could conduct this experimental trials to get very accurate results.

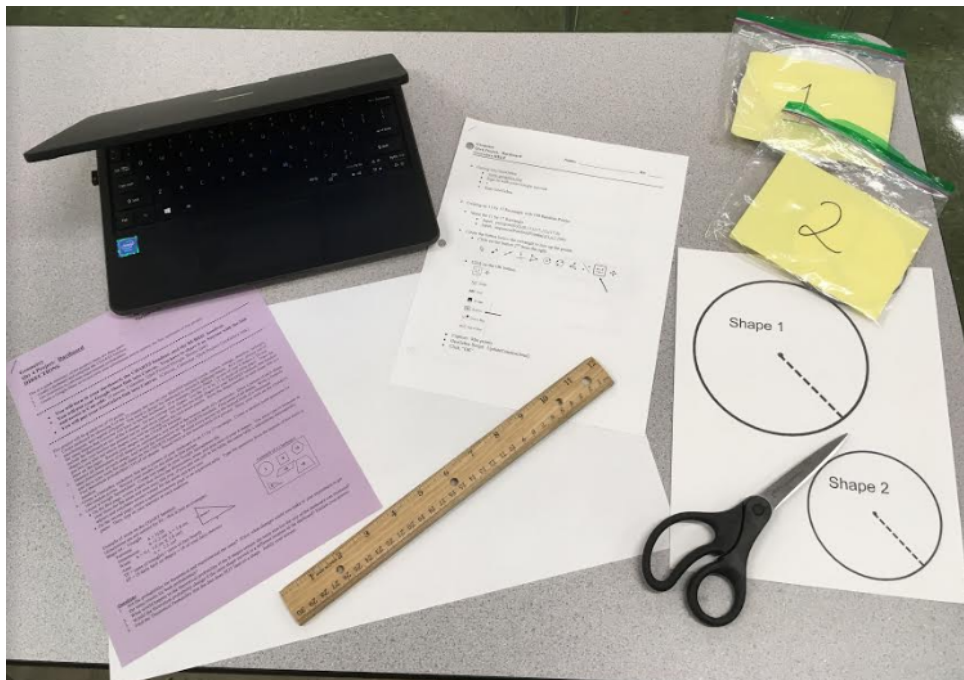


	in	out	color
①			purple
②			blue
③			green
④			red
⑤			brown
⑥			pink
⑦			yellow
⑧			orange
⑨			gray
⑩			black

	in	out	color
①	2	8	purple
②	3	7	blue
③	6	4	green
④	3	7	red
⑤	3	7	brown
⑥	4	6	pink
⑦	6	4	yellow
⑧	3	7	orange
⑨	5	5	gray
⑩	4	6	black
Total:	39	61	100

Materials

- The teacher will make copies of the project, one for each student.
 - Project Packet (print on colored paper – students will turn in this packet)
 - Dartboard DIRECTIONS
 - Dartboard CHARTS
 - Dartboard RUBRIC
 - Helps Packet (print on white paper – students keep this packet as a reference packet)
 - GeoGeobra HELPS sheet
 - Google Sheets HELPS sheet
- 11 inches by 17 inches white paper (This will be the dartboard. Students will trace 6 of the big shapes on this dartboard. Students will work with pairs, so have enough paper for each group)
- Rulers (in centimeters)
- Scissors (unless the teacher wants to cut the shapes out of the cardstock paper before the students do the activity)
- Computer or Chromebook
- The teacher will make about 10 sets of the big shapes with White Cardstock Paper.
- OPTIONAL: It is helpful to have the cut-out shapes organized in plastic bags and place them around the room so that students have room to select their shapes.



Geometry

Project: Dartboard

DIRECTIONS

Name: _____ Hr: _____

Partner's Name: _____

This is a quick summary of the project; there are three parts:

1. Create a dartboard and complete the CHARTS handout.
2. Use GeoGebra to calculate experimental probabilities.
3. Create a Google Sheet to record your area calculations and to answer the four questions of this project.

-
- You will turn in your dartboard and the purple packet.
 - You will share your Google Sheet link with your teacher. You have to share it as Anyone with the link and access is Can edit.
 - You will share your GeoGebra link with your teacher.
-

These are the steps for the Dartboard Project:

1. Create dartboard by choosing 6 of the 22 shapes that are on your SHAPES handout (circle, square, triangle, rhombus, rectangle, parallelogram, and regular polygon). You need to design the dartboard to include 6 different shapes. As a class, choose one of the same shapes that everyone will have in his/her dartboard. (For example, choose Shape 1 as one of your 6 shapes.) Do not include the dartboard as one of the 6 shapes. Do not overlap your shapes. Use the 11 in by 17 in sheet of white paper. Trace the shapes that your teacher has printed out on card stock paper. Do not use the small shapes on your SHAPES handout; that handout just lets you plan out which shapes you will be tracing from the card stock papers. Write the shape number in each shape that you draw on the paper. Record your shapes on the CHARTS handout.
2. Measure all dimensions of all shapes in centimeters, and round to the nearest tenth of a centimeter. Draw a picture of each shape, label dimensions on each shape, and record the dimensions on the CHARTS handout. Find areas of all shapes. Write a formula and show your work for each shape on the CHARTS handout. Round every answer to the nearest tenth of a centimeter.
3. Find theoretical probabilities (TP) of all shapes. To find the TP, find the area of each shape as a percentage of the total area of the dartboard. Round each to the nearest tenth of a percent. Show your work on the CHARTS handout.
4. Create a GeoGebra worksheet that has a picture of your dartboard in an 11 by 17 rectangle. Create a random point generator for 100 random points. Record your results on the Experimental Trials Chart in the CHARTS handout. You will record five trials. (Since any one trial can have unusual results, it is necessary to perform more than one trial.)
5. Create a Google Spreadsheet. Create two pages in your Google Spreadsheet file.
 - On the first page, create a table that shows the dimensions, areas, and area sum of your 6 shapes. You must type in formulas in your sheet for the areas and area sum; this way, if you edit one cell in the table, the entire table's calculations automatically adjusts and calculates correct values for the areas and area sum.
 - On the second page, create a question/answer page in an organized table. Type the questions from the bottom of this sheet of paper. Then, type in your answers to each question.

Example of work on the CHART handout:

(dimensions are not correct for #4 – this is just an example)

Shape #4 – Triangle

Formula: $A = \frac{1}{2}bh$

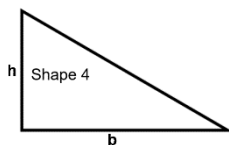
Dimensions: $b = 1.2 \text{ cm}$; $h = 3.8 \text{ cm}$

Work: $A = \frac{1}{2} (1.2 \text{ cm}) (3.8 \text{ cm})$

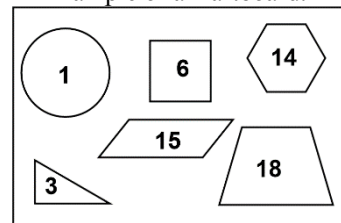
Area: $A = 2.3 \text{ cm}^2$

TP = (area of triangle) \div (area of dart board)

EP = (# darts land on shape) \div (# of total darts thrown)



Example of a Dartboard:



Questions

1. Are the probabilities for theoretical and experimental the same? If not, what changes could you make to your experiment to get the same results for both probabilities?
2. What would happen to the theoretical probability if the 6 shapes remain the same size but the size of the dartboard was increased?
3. Would the theoretical probability change if the same shape moved to a different location of the dartboard? Explain your answer.
4. Find the Theoretical Probability that the dart does NOT land on a shape. Justify your answer.

Geometry

Project: Dartboard

CHARTS

Name: _____ **Hour:** _____

Partner's Name: _____

Area of the Dartboard

Work to find the area of the dartboard:

Experimental Trials Chart

	Shape A #	Shape B #	Shape C #	Shape D #	Shape E #	Shape F #	Misses	TOTAL
Trial 1								= 100
Trial 2								= 100
Trial 3								= 100
Trial 4								= 100
Trial 5								= 100

Shape C # _____ Shape Name: _____

Dimensions (labeled picture and measurements)

Area (formula and work)

TP =

EP =

Shape D # _____ Shape Name: _____

Dimensions (labeled picture and measurements)

Area (formula and work)

TP =

EP =

Geometry

/45 pts

Name: _____

Partner's Name: _____ Hour: _____

	5 - Excellent	3 - Fair	1 - Poor
Dartboard Design	Dartboard displays all 4 of the following: 1. 6 distinct shapes, not overlapping 2. Shapes with correct numbers 3. A pentagon or a hexagon is one of the 6 distinct shapes 4. Professional display	Dartboard displays all 3 of the following: 1. 6 distinct shapes, not overlapping 2. Shapes with correct numbers 3. A pentagon or a hexagon is one of the 6 distinct shapes 4. Professional display	Dartboard displays all 1-2 of the following: 1. 6 distinct shapes, not overlapping 2. Shapes with correct numbers 3. A pentagon or a hexagon is one of the 6 distinct shapes 4. Professional display
Dimensions of Shapes	Shapes are correctly identified, sides are accurately measured, and correct units are used (centimeters) on the paper.	Most shapes are correctly identified, sides are accurately measured, and correct units are used. There are minimal errors.	Less than half of the shapes are correctly identified, sides are accurately measured, and correct units are used. There are many errors.
Area Calculations	Correct formulas are used for every shape, work is shown, and all calculations and units are correct on the paper.	Correct formulas are used for most shapes, work is shown, and all calculations and units are correct. There may be minimal errors.	Correct formulas are used for less than half of the shapes, work is shown, and all calculations and units are correct. There are many errors.
Theoretical Probability (TP) Calculated	The theoretical probabilities calculated are correct for each shape. Work is shown for all calculation on the paper.	The theoretical probabilities calculated are correct for most shapes. All work is not shown. There may be minimal errors.	The theoretical probabilities calculated are correct for less than half of the shapes. All work is not shown. There are many errors.
Experimental Probability (EP) Calculated	The experimental probabilities calculated are correct for each shape. Work is shown for all calculations. Experimental Trials Chart is completed on the paper.	The experimental probabilities calculated are correct for most shapes. All work and trials are not shown. There may be minimal errors.	The experimental probabilities calculated are correct for less than half of the shapes. All work and trials are not shown. There are many errors.
GeoGebra link	The GeoGebra link is in Canvas and works.		
GeoGebra worksheet	All of the following are met: 1. Dartboard picture is in GeoGebra 2. Dartboard is inscribed in the rectangle 3. Random Point generator works	Most of the following are met: 1. Dartboard picture is in GeoGebra 2. Dartboard is inscribed in the rectangle 3. Random Point generator works	One/some of the following are met: 1. Dartboard picture is in GeoGebra 2. Dartboard is inscribed in the rectangle 3. Random Point generator works
Answers to Questions	All of the following are met: 1. All four questions are answered correctly and completely. 2. Answers use complete sentences.	Most of the following are met: 1. All four questions are answered correctly and completely. 2. Answers use complete sentences.	One/some of the following are met: 1. All four questions are answered correctly and completely. 2. Answers use complete sentences.
Google Sheets (followed directions)	All of the following are met: 1. One Google Sheet file, each labeled 2. 1 st sheet is for the calculations 3. 2 nd sheet is for the answers to the ?s 4. The file is submitted into Canvas.	Most of the following are met: 1. One Google Sheet file, each labeled 2. 1 st sheet is for the calculations 3. 2 nd sheet is for the answers to the ?s 4. The file is submitted into Canvas.	One/some of the following are met: 1. One Google Sheet file, each labeled 2. 1 st sheet is for the calculations 3. 2 nd sheet is for the answers to the ?s 4. The file is submitted into Canvas.
Google Sheets (organization)	All of the following are met: 1. A grid is shown in both sheets. 2. Data is centered in each cell. 3. All information can be read without scrolling to the right. 4. Colors are used in both sheets.	Most of the following are met: 1. A grid is shown in both sheets. 2. Data is centered in each cell. 3. All information can be read without scrolling to the right. 4. Colors are used in both sheets.	One/some of the following are met: 1. A grid is shown in both sheets. 2. Data is centered in each cell. 3. All information can be read without scrolling to the right. 4. Colors are used in both sheets.
Google Sheets (accurate math)	The formulas are correct for area and for total sum. Work is shown on packet (lower 3 rd side)	The formulas are not all correct for area and for total sum and work is shown in packet.	The formulas are not all correct for area and for total sum and work is shown in packet.

Project: Dartboard RUBRIC

Turn in: (1) dartboard, (2) purple packet, and (3) Google Sheets link

	5 - Excellent	3 - Fair	1 - Poor
Dartboard Design	Dartboard displays all 4 of the following: 1. 6 distinct shapes, not overlapping 2. Shapes with correct numbers 3. A pentagon or a hexagon is one of the 6 distinct shapes 4. Professional display	Dartboard displays all 3 of the following: 1. 6 distinct shapes, not overlapping 2. Shapes with correct numbers 3. A pentagon or a hexagon is one of the 6 distinct shapes 4. Professional display	Dartboard displays all 1-2 of the following: 1. 6 distinct shapes, not overlapping 2. Shapes with correct numbers 3. A pentagon or a hexagon is one of the 6 distinct shapes 4. Professional display
Dimensions of Shapes	Shapes are correctly identified, sides are accurately measured, and correct units are used (centimeters) on the paper.	Most shapes are correctly identified, sides are accurately measured, and correct units are used. There are minimal errors.	Less than half of the shapes are correctly identified, sides are accurately measured, and correct units are used. There are many errors.
Area Calculations	Correct formulas are used for every shape, work is shown, and all calculations and units are correct on the paper.	Correct formulas are used for most shapes, work is shown, and all calculations and units are correct. There may be minimal errors.	Correct formulas are used for less than half of the shapes, work is shown, and all calculations and units are correct. There are many errors.
Theoretical Probability (TP) Calculated	The theoretical probabilities calculated are correct for each shape. Work is shown for all calculation on the paper.	The theoretical probabilities calculated are correct for most shapes. All work is not shown. There may be minimal errors.	The theoretical probabilities calculated are correct for less than half of the shapes. All work is not shown. There are many errors.
Experimental Probability (EP) Calculated	The experimental probabilities calculated are correct for each shape. Work is shown for all calculations. Experimental Trials Chart is completed on the paper.	The experimental probabilities calculated are correct for most shapes. All work and trials are not shown. There may be minimal errors.	The experimental probabilities calculated are correct for less than half of the shapes. All work and trials are not shown. There are many errors.
Answers to Questions	All of the following are met: 1. All four questions are answered correctly and completely. 2. Answers use complete sentences.	Most of the following are met: 1. All four questions are answered correctly and completely. 2. Answers use complete sentences.	One/some of the following are met: 1. All four questions are answered correctly and completely. 2. Answers use complete sentences.
Google Sheets (followed directions)	All of the following are met: 1. One Google Sheet file, each labeled 2. 1 st sheet is for the calculations 3. 2 nd sheet is for the answers to the ?s 4. The file is submitted into Canvas.	Most of the following are met: 1. One Google Sheet file, each labeled 2. 1 st sheet is for the calculations 3. 2 nd sheet is for the answers to the ?s 4. The file is submitted into Canvas.	One/some of the following are met: 1. One Google Sheet file, each labeled 2. 1 st sheet is for the calculations 3. 2 nd sheet is for the answers to the ?s 4. The file is submitted into Canvas.
Google Sheets (organization)	All of the following are met: 1. A grid is shown in both sheets. 2. Data is centered in each cell. 3. All information can be read without scrolling to the right. 4. Colors are used in both sheets.	Most of the following are met: 1. A grid is shown in both sheets. 2. Data is centered in each cell. 3. All information can be read without scrolling to the right. 4. Colors are used in both sheets.	One/some of the following are met: 1. A grid is shown in both sheets. 2. Data is centered in each cell. 3. All information can be read without scrolling to the right. 4. Colors are used in both sheets.
Google Sheets (accurate math)	The formulas are correct for area and for total sum. Work is shown on packet (lower 3 rd side)	The formulas are not all correct for area and for total sum and work is shown in packet.	The formulas are not all correct for area and for total sum and work is shown in packet.

➤ Getting into GeoGebra

- www.geogebra.org
- Sign In with your Google account
- +
- Start GeoGebra

➤ Creating an 11 by 17 Rectangle with 100 Random Points

- Make the 11 by 17 Rectangle
 - Input: `polygon((0,0),(0,11),(17,11),(17,0))`
 - Input: `sequence(RandomPointIn(q1),a,1,100)`
- Create the button below the rectangle to mix up the points
 - Click on the button 2nd from the right.



- Click on the OK button.



- Click just below the x-axis, underneath the rectangle on the graph.
- Caption: Mix points
- GeoGebra Script: `UpdateConstruction()`
- Click, "OK"

➤ Placing Your Dartboard Picture in the GeoGebra 11 by 17 Rectangle

- Click on the button 2nd from the right.



- Click on “Image”



- Choose your file that has the picture of your Dartboard.

➤ Saving Your GeoGebra File

- Title: Dartboard Project First and Last Name
- Shared

➤ Share your GeoGebra File Link with your teacher

- Share

Google Sheets HELPS

SAMPLE: Project DARTBOARD

File Edit View Insert Format Data Tools Add-ons Help [All changes saved in Drive](#)

	A	B	C	D	E	F	G	H	I	J	K
1	Shape Number	Shape Name	r	b	h	n	b1	b2	d1	d2	Area
2	1	Circle	7								153.93804
3	7	Triangle		13.5	11.3						76.275
4	11	Pentagon		8.6	6.4	5					137.6
5	16	Parallelogram		13.8	8.5						117.3
6	17	Trapezoid			4.9		10	16.7			65.415
7	21	Kite							14.4	9.7	69.84
8											
9											
10											
11		3.141592654									
12											
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31											
32											

fx

+ = Calculations Answers to Questions



SAMPLE: Project DARTBOARD



File Edit View Insert Format Data Tools Add-ons Help [All changes saved](#)



100% ▾



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.00

123 ▾

Arial ▾

10 ▾



fx

A

1

Answers to Questions:

2

(1) Are the probabilities for theoretical and experimental the same? If not, what changes could you make to your experiment to get the same results for both probabilities?

3

This would be my answer to question #1. Remember to use complete sentences.

4

5

(2) What would happen to the theoretical probability if the 6 shapes remain the same size but the size of the dartboard was increased?

6

This would be my answer to question #2. Remember that before turning in your project, you should talk with other people to make sure that your answers are correct. Talk with your teacher also about your project before turning it in.

7

8

(3) Would the theoretical probability change if the same shape was moved to a different location of the dartboard? Explain your answer.

9

This would be my answer to question #3. Remember that the goal for this project is to: learn, have fun, understand, and get a great score!

10

11

(4) Find the Theoretical Probability that the dart does NOT land on a shape. Justify your answer.

12

This would be my answer to question #4. Remember to ask other people for help if you need help, but don't ask your teacher (your boss) until you have asked other students (your colleagues) first. You don't want to annoy your boss!

13

14

***TIP: Enter newlines in cells by using Ctrl+Enter while editing the cell.**

15

16

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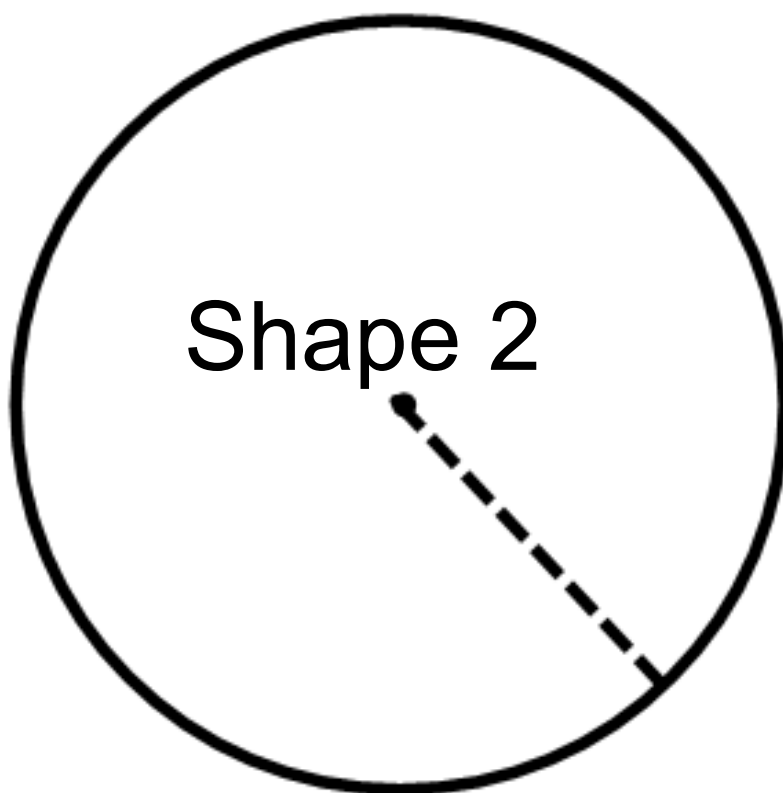
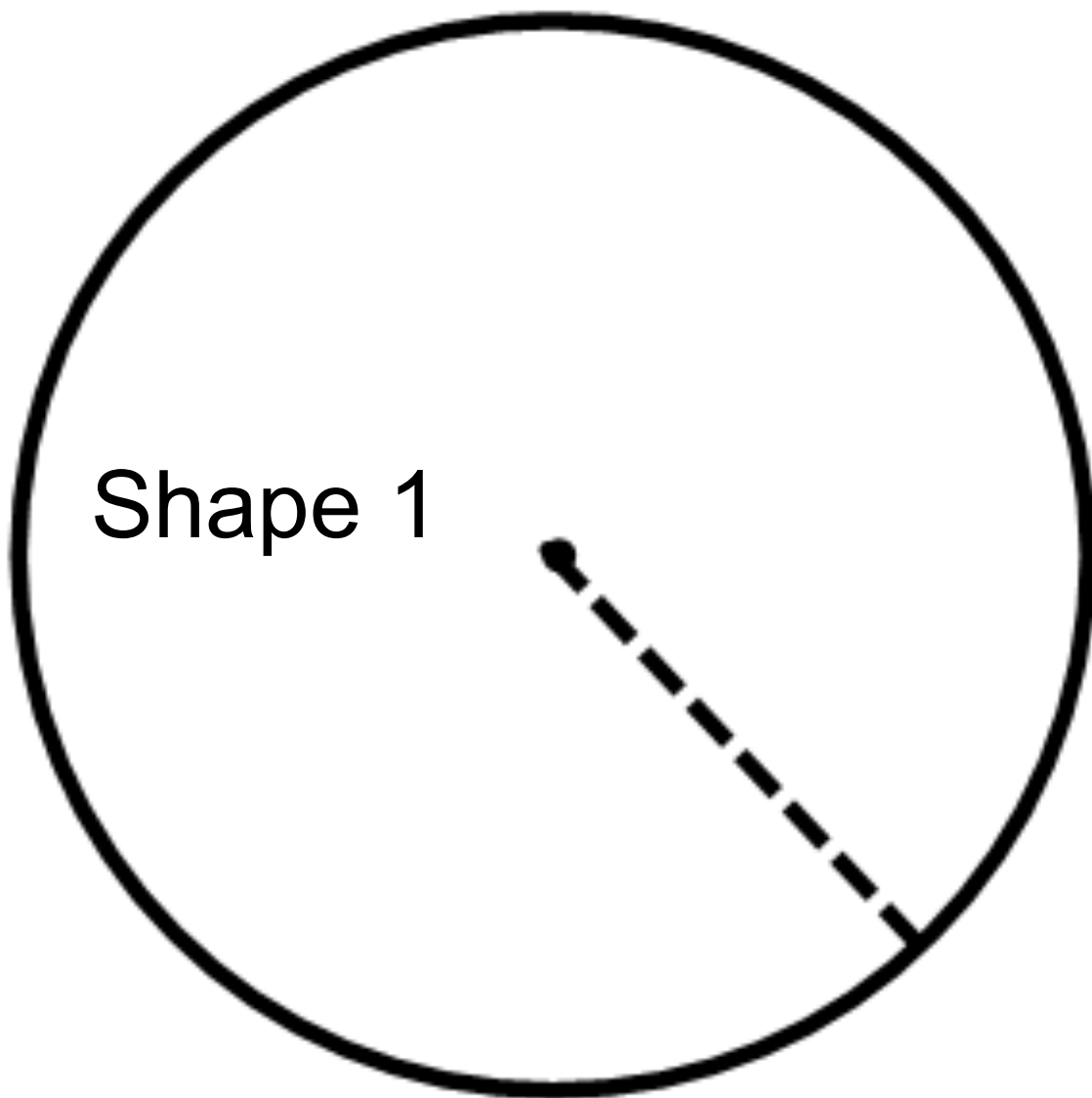
22

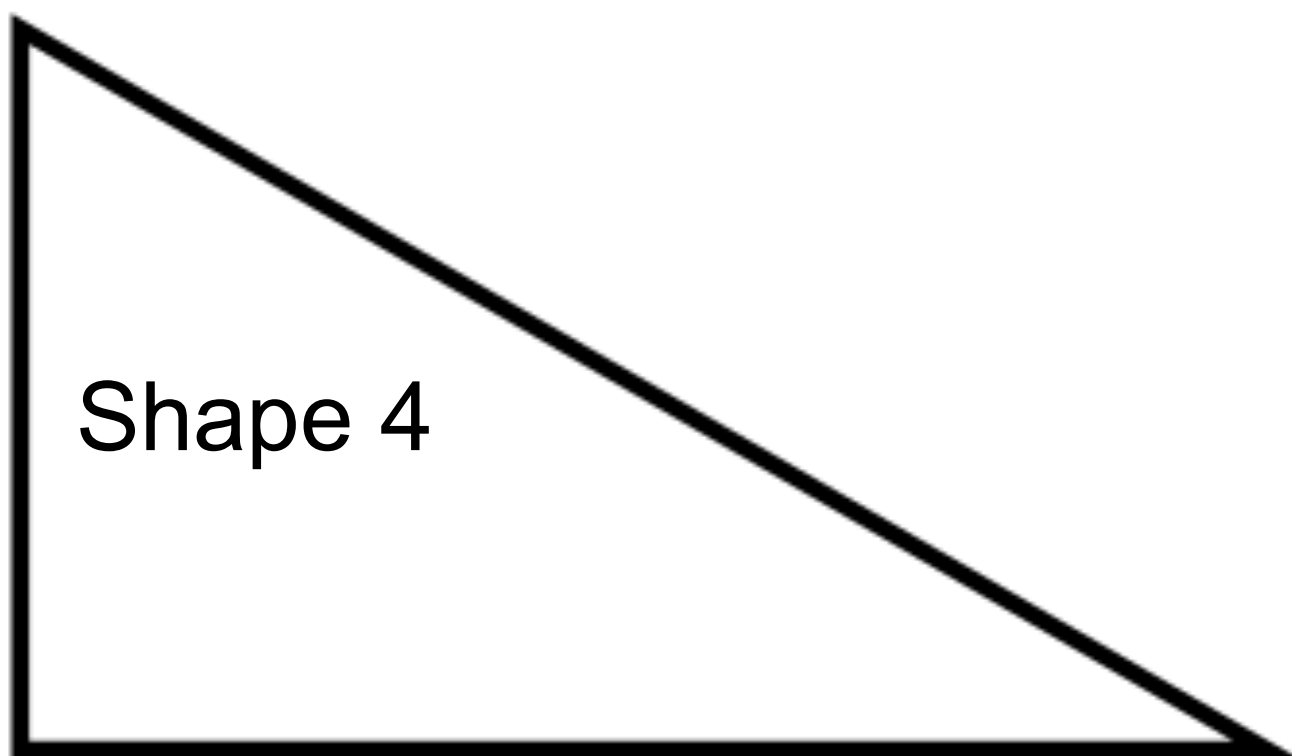
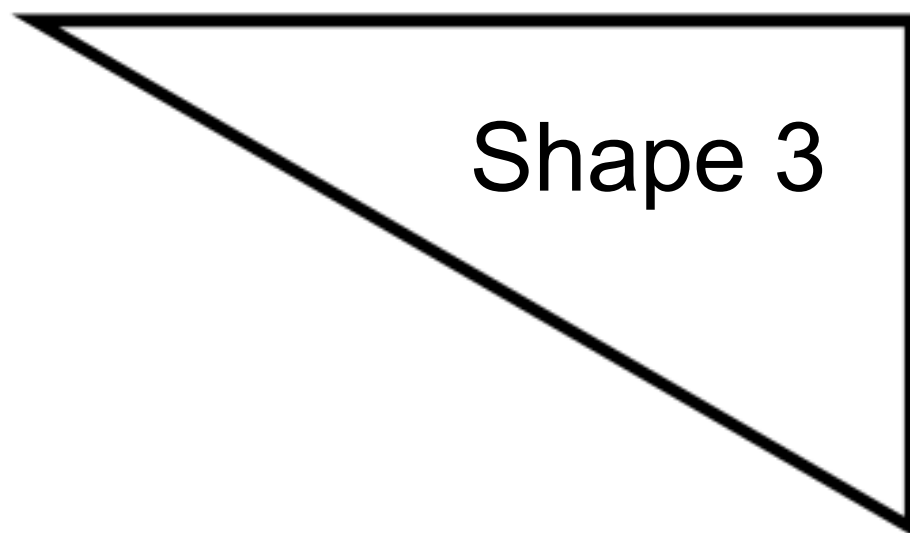
23



Calculations ▾

Answers to Questions ▾



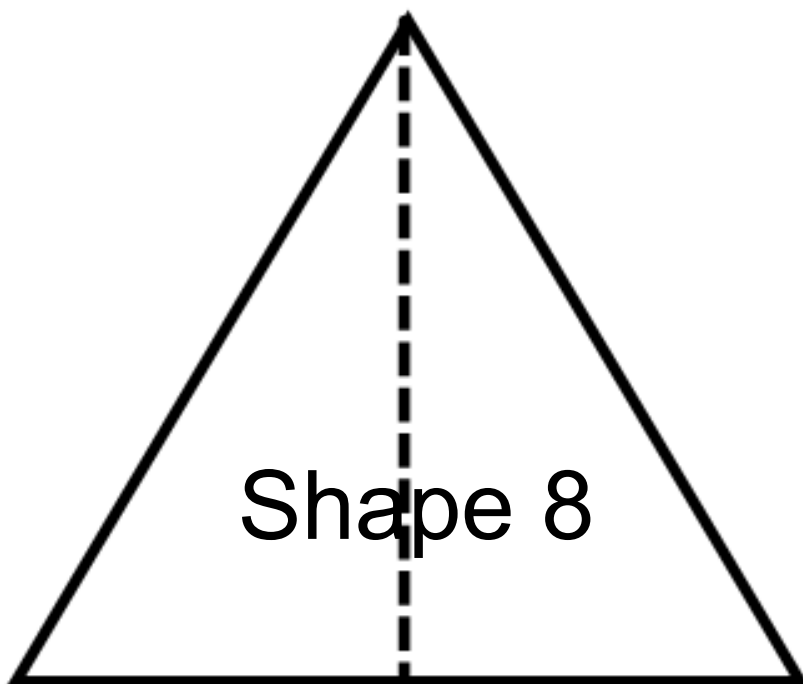
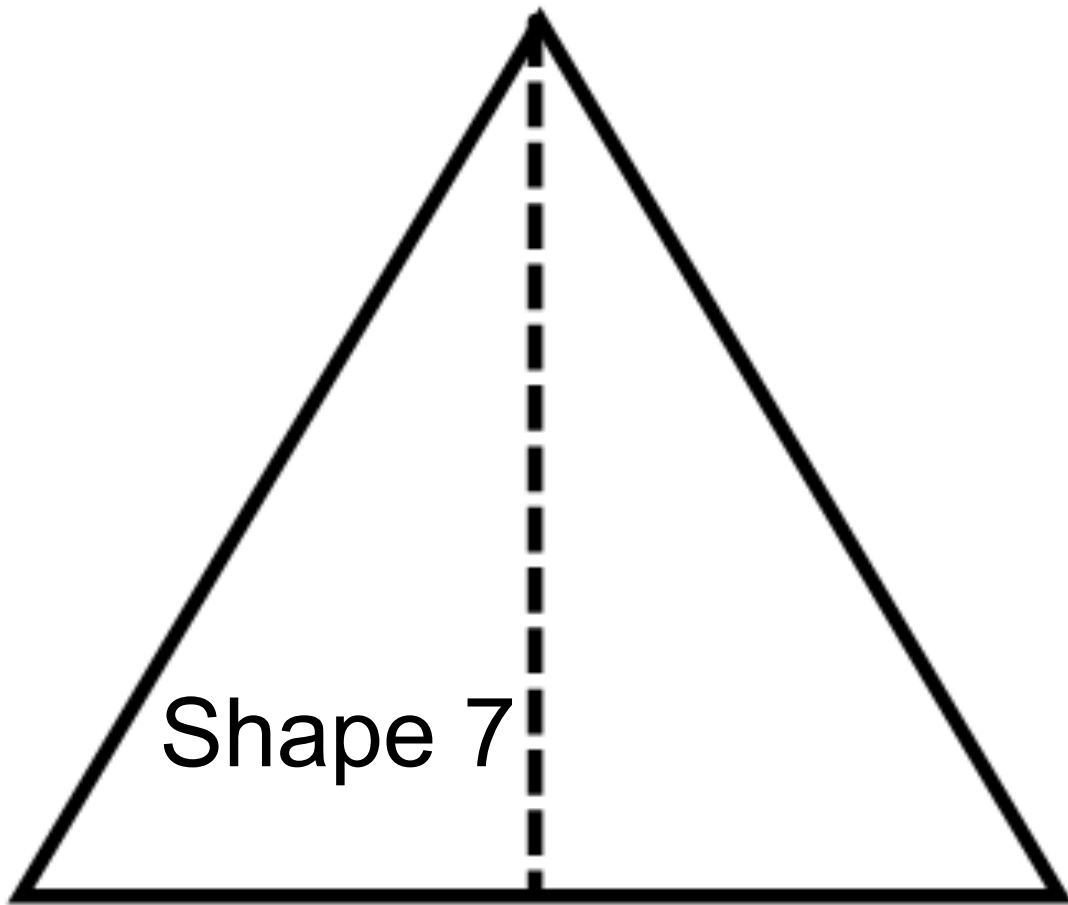


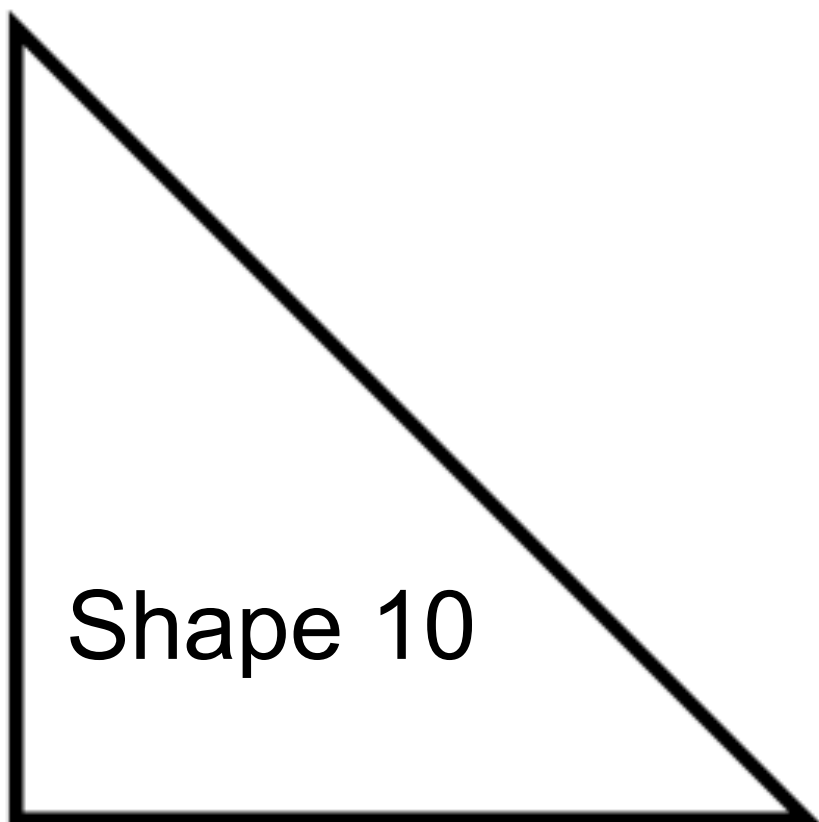
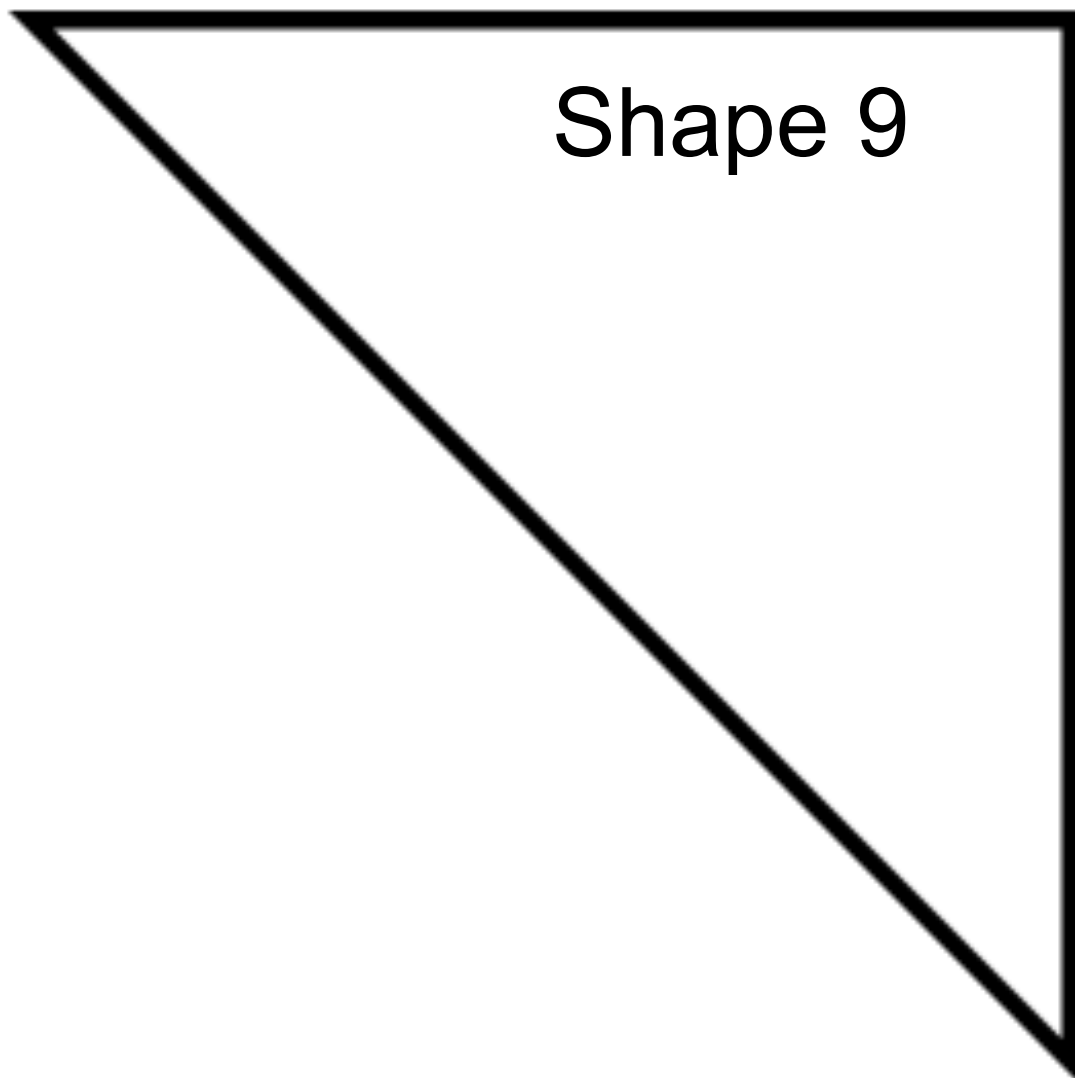


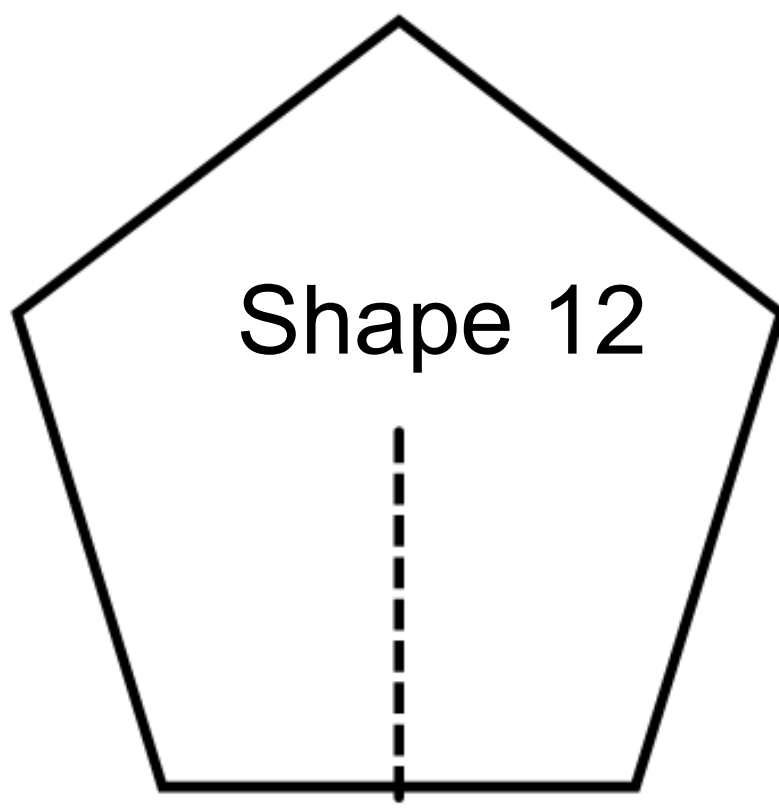
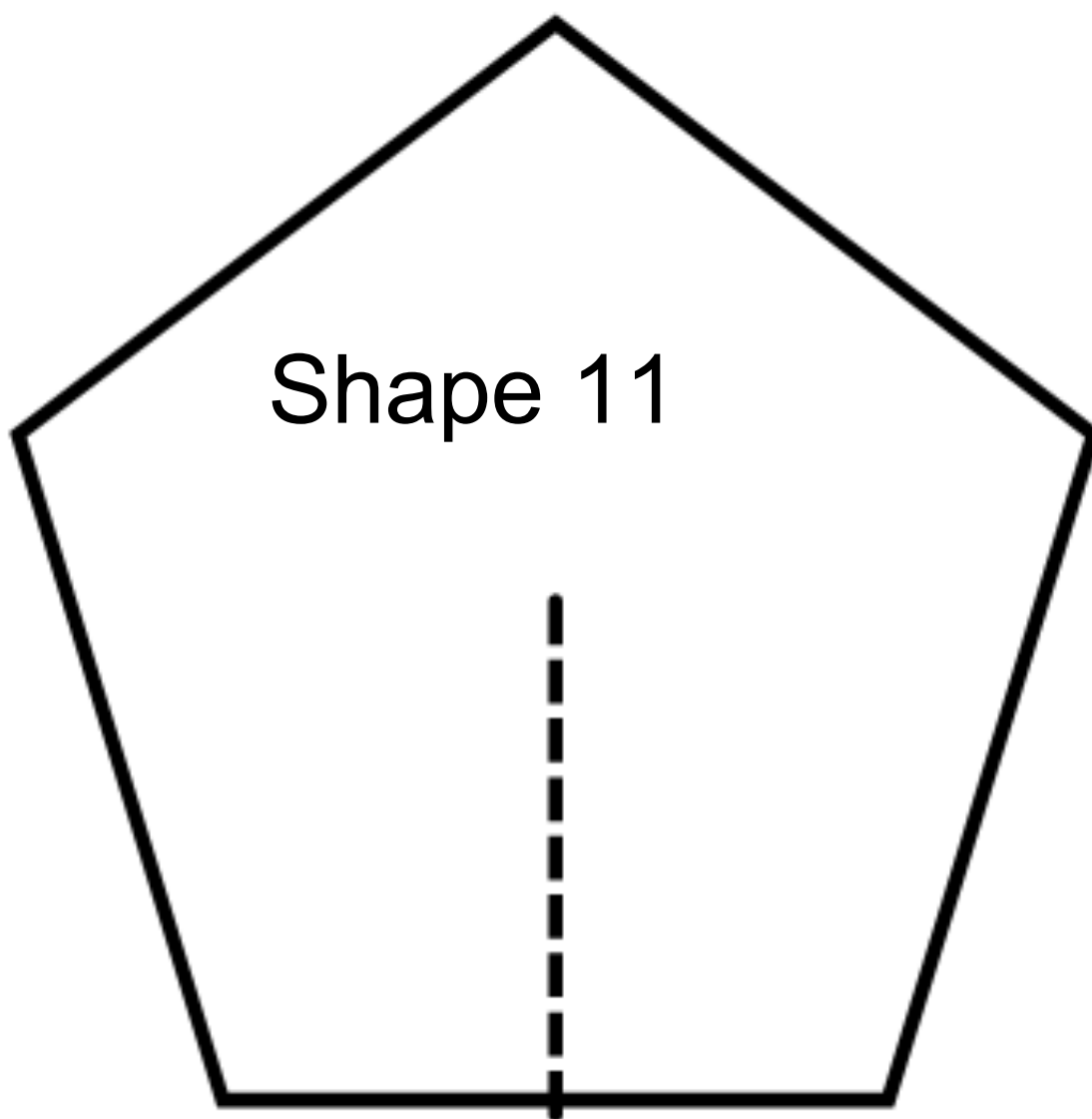
Shape 5

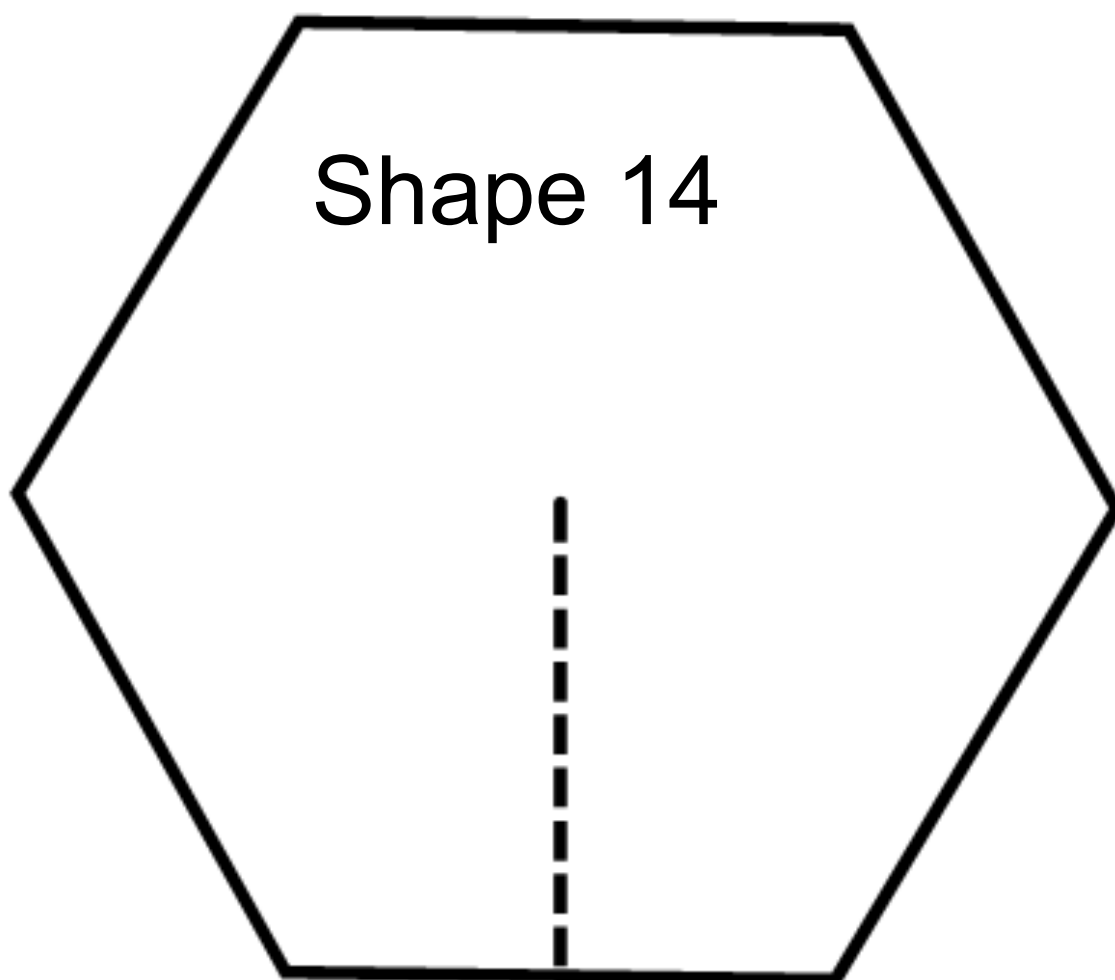
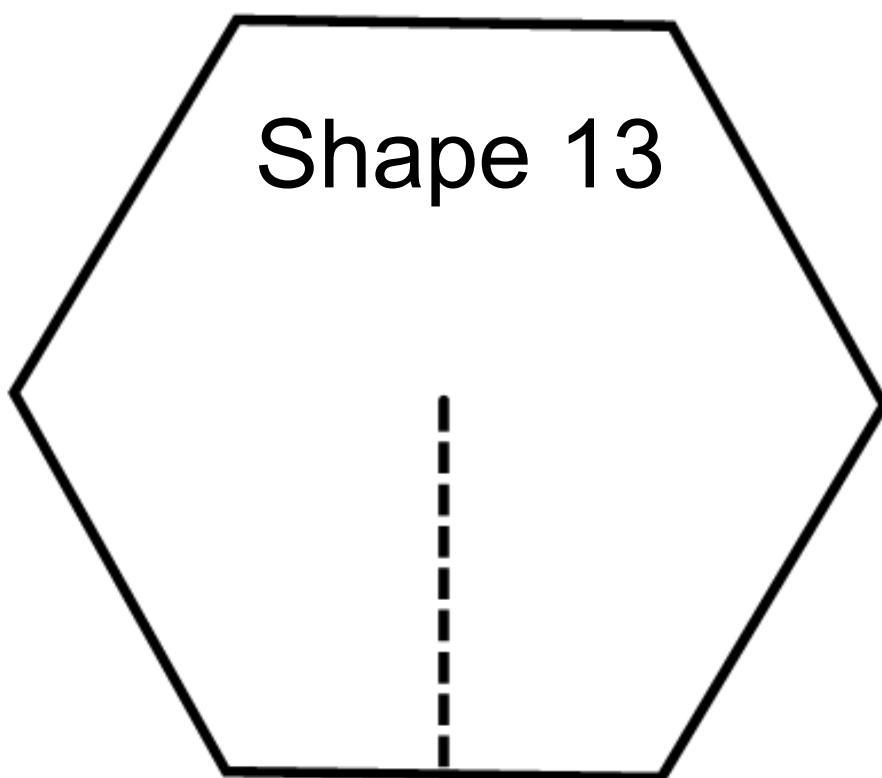


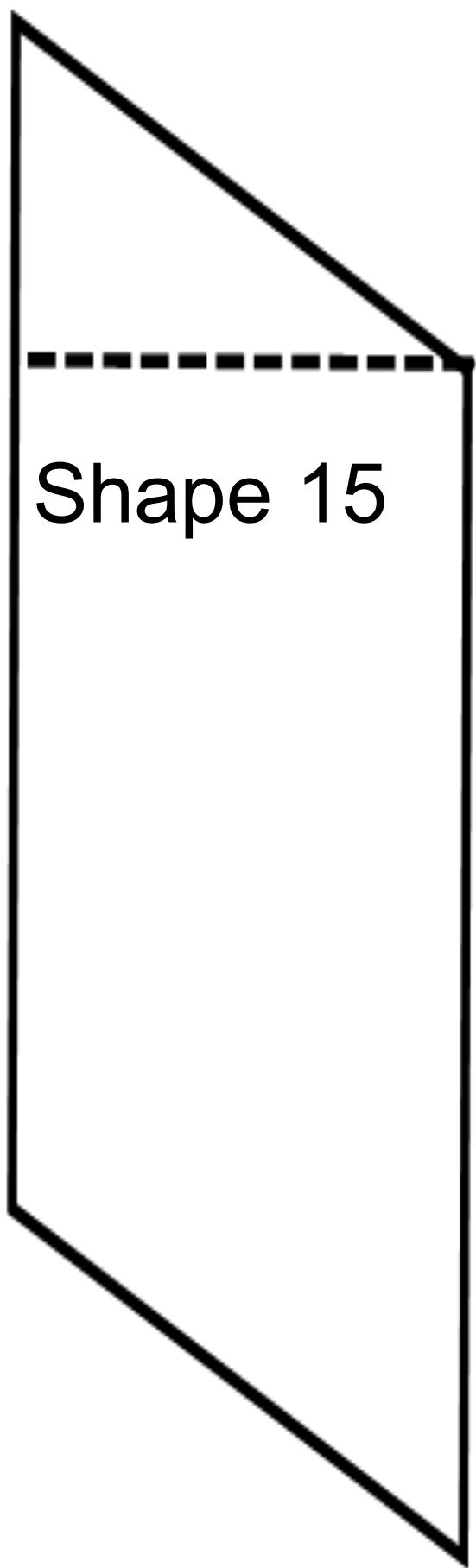
Shape 6



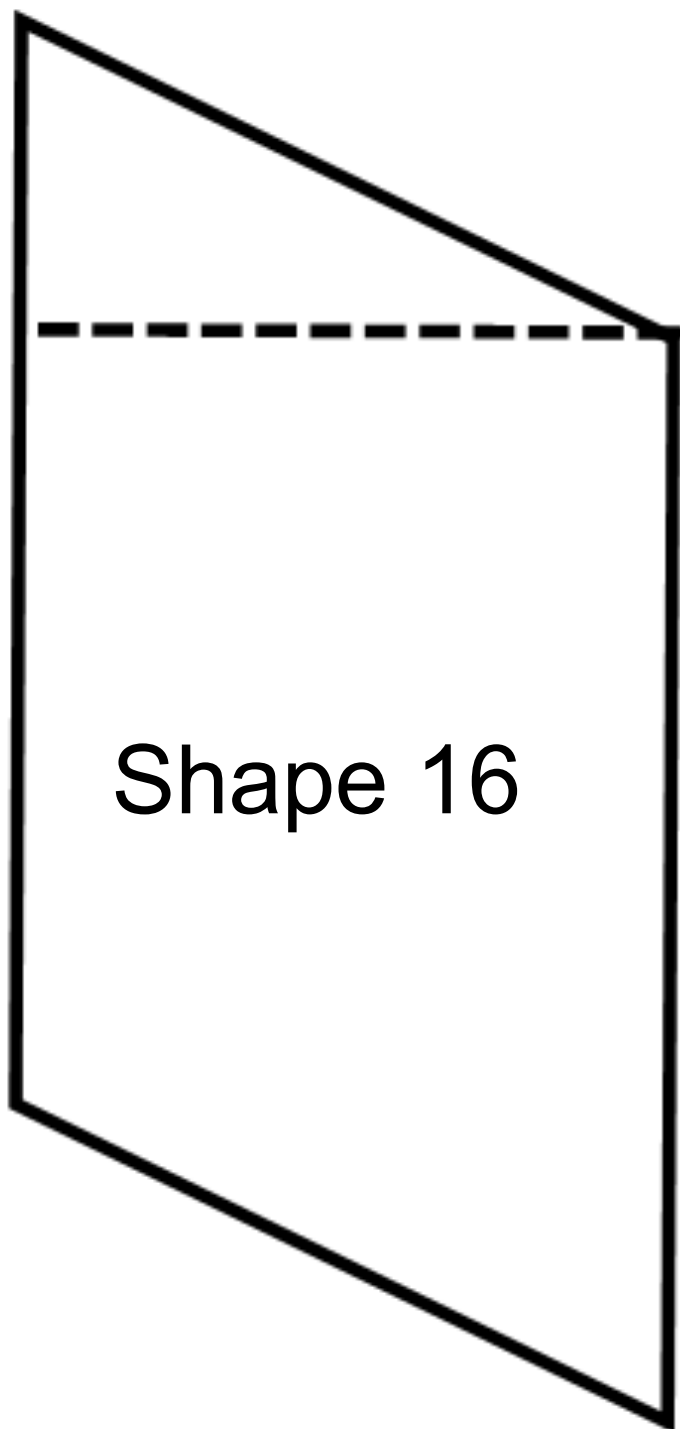




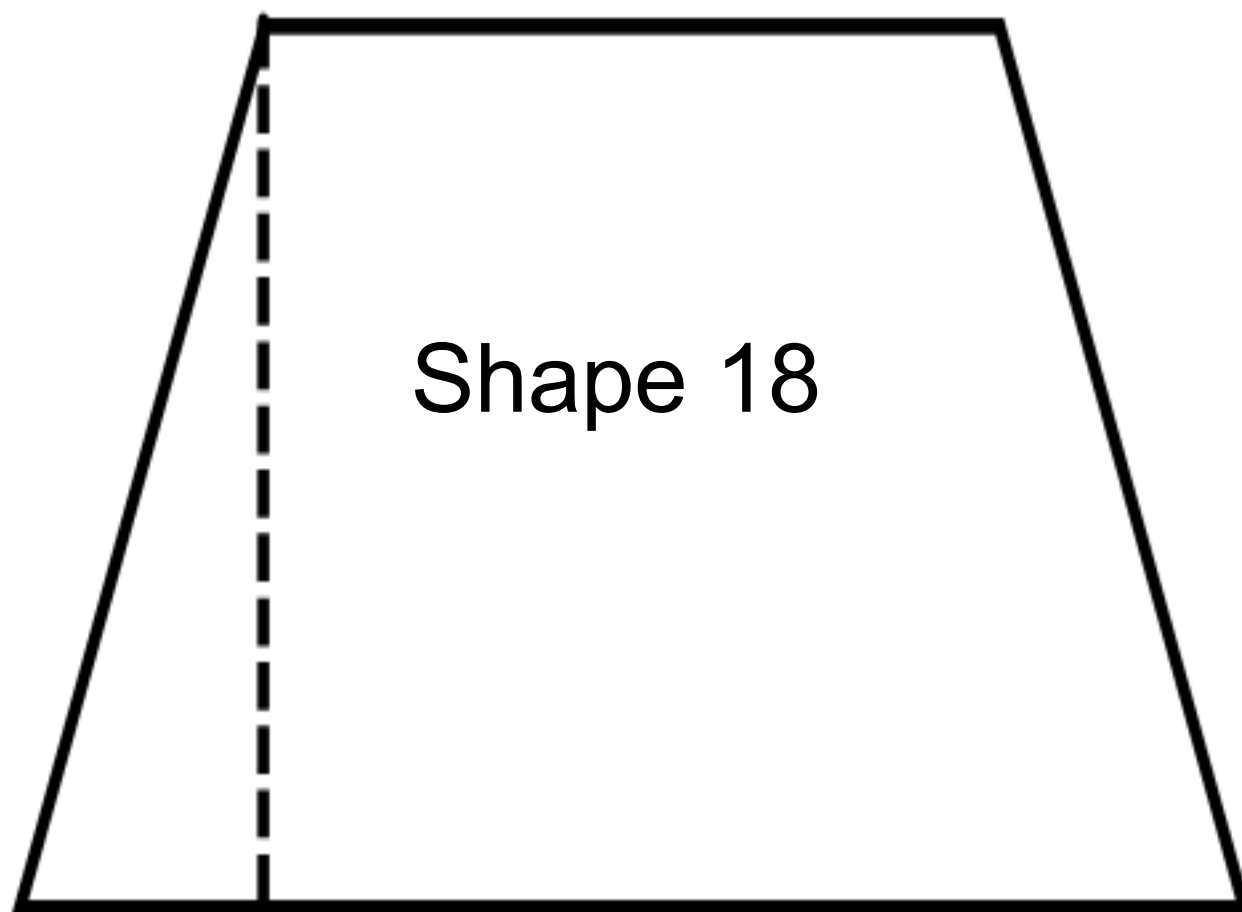
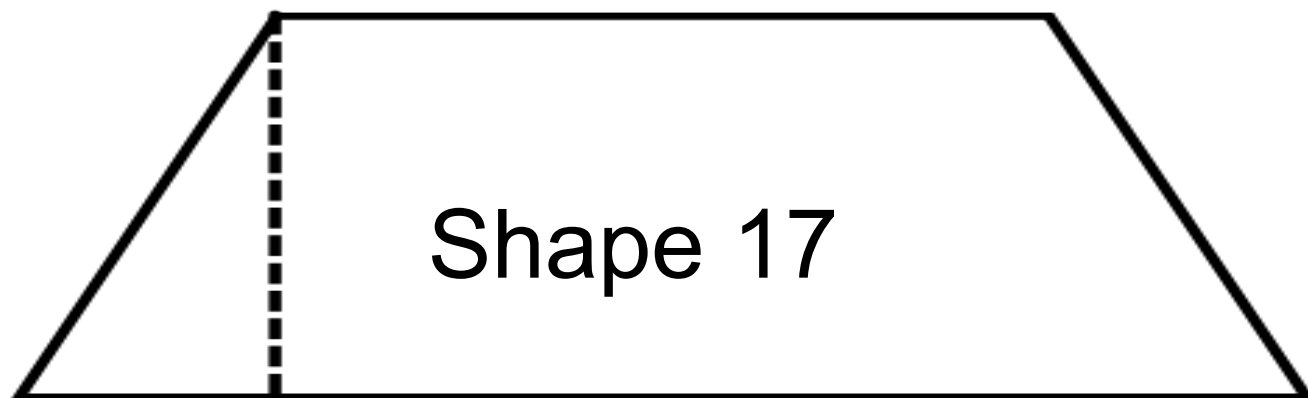




Shape 15

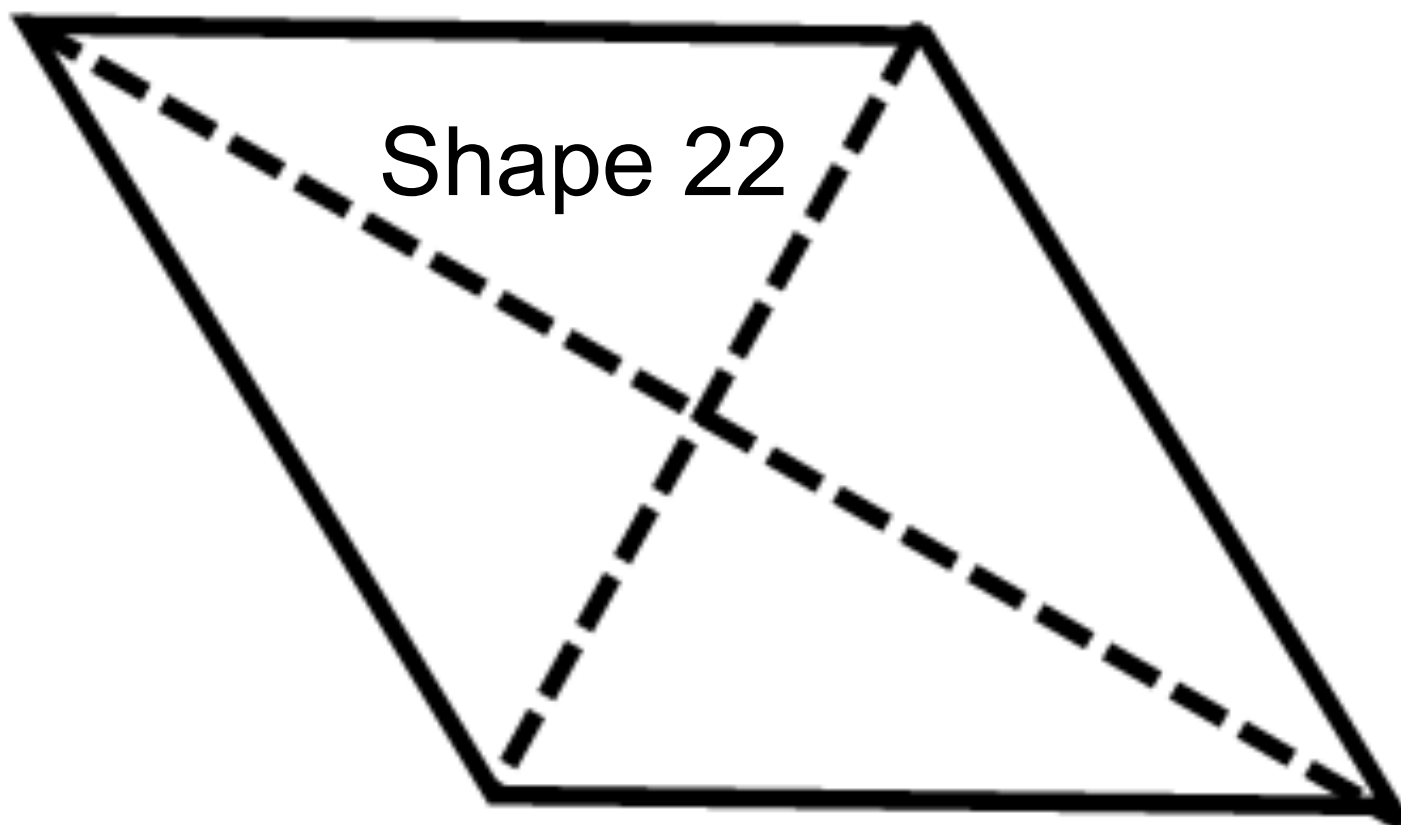
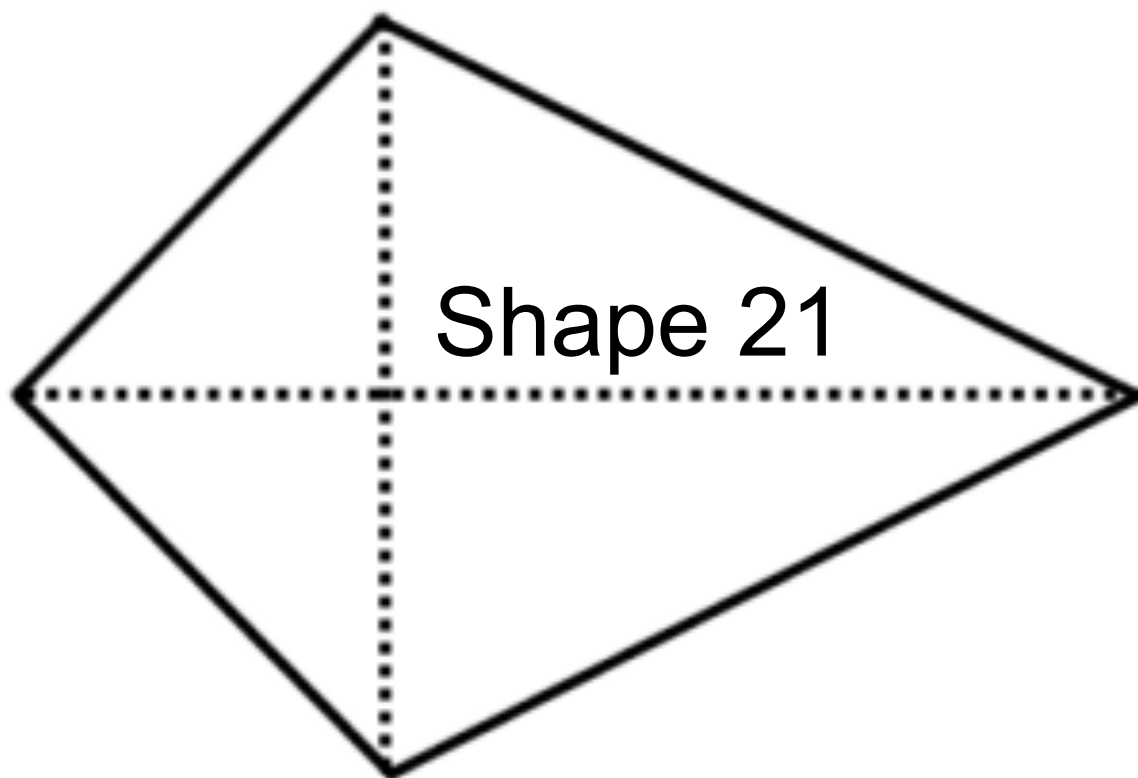


Shape 16



Shape 19

Shape 20



Geometry
Quarter 4 Project: Dartboard
SOLUTIONS to the Charts

Area of the Dartboard

Work to find the area of the dartboard:



$$\frac{11in}{1} \frac{2.54cm}{1in} \approx 27.9cm$$

$$\frac{17in}{1} \frac{2.54cm}{1in} \approx 43.2cm$$

$$A = bh \approx (27.9cm)(43.2cm) \approx 1,205.3cm^2$$

**Measure to the insides of the sides of each shape.*
**Measure to the center of a point.*

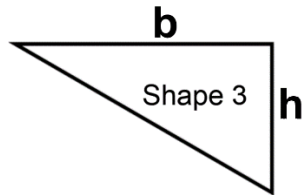
Measurements/Work Chart

<p>Shape 1 Shape Name: Circle</p> <p>Dimensions (labeled picture and measurements)</p> <p>$r \approx 7.1\text{ cm}$</p> <p>Area (formula and work)</p> <p>$A = \pi r^2$</p> <p>$A = \pi(7.1\text{ cm})^2$</p> <p>$A \approx 158.4\text{ cm}^2$</p> <p>TP =</p> <p>EP =</p>	<p>Shape 2 Shape Name: Circle</p> <p>Dimensions (labeled picture and measurements)</p> <p>$r \approx 5.0\text{ cm}$</p> <p>Area (formula and work)</p> <p>$A = \pi r^2$</p> <p>$A = \pi(5.0\text{ cm})^2$</p> <p>$A \approx 78.5\text{ cm}^2$</p> <p>TP =</p> <p>EP =</p>
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Shape 3 **Shape Name:** Right Triangle

Dimensions (labeled picture and measurements)

$b \approx 11.3 \text{ cm}$
 $h \approx 6.5 \text{ cm}$



Area (formula and work)

$$A = \frac{1}{2} bh$$

$$A \approx \frac{1}{2} (11.3 \text{ cm})(6.5 \text{ cm})$$

$$A \approx 36.7 \text{ cm}^2$$

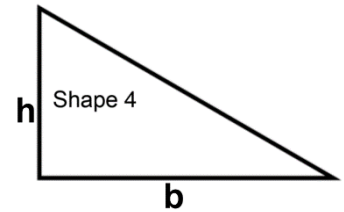
TP =

EP =

Shape 4 **Shape Name:** Right Triangle

Dimensions (labeled picture and measurements)

$b \approx 16.1 \text{ cm}$
 $h \approx 9.3 \text{ cm}$



Area (formula and work)

$$A = \frac{1}{2} bh$$

$$A \approx \frac{1}{2} (16.1 \text{ cm})(9.3 \text{ cm})$$

$$A \approx 74.9 \text{ cm}^2$$

TP =

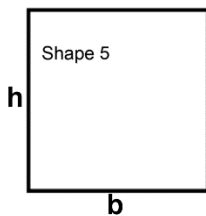
EP =

Shape 5 **Shape Name:** Square

Dimensions (labeled picture and measurements)

$$b \approx 12.3 \text{ cm}$$

$$h \approx 12.3 \text{ cm}$$



Area (formula and work)

$$A = bh$$

$$A \approx (12.3 \text{ cm})(12.3 \text{ cm})$$

$$A \approx 151.3 \text{ cm}^2$$

TP =

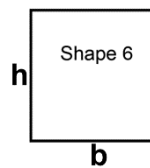
EP =

Shape 6 **Shape Name:** Square

Dimensions (labeled picture and measurements)

$$b \approx 8.3 \text{ cm}$$

$$h \approx 8.3 \text{ cm}$$



Area (formula and work)

$$A = bh$$

$$A \approx (8.3 \text{ cm})(8.3 \text{ cm})$$

$$A \approx 68.9 \text{ cm}^2$$

TP =

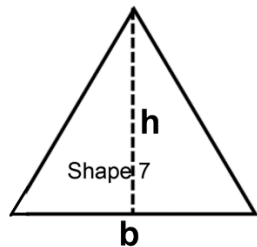
EP =

Shape 7 **Shape Name:** Triangle (Isosceles, non-right)

Dimensions (labeled picture and measurements)

$$b \approx 13.5 \text{ cm}$$

$$h \approx 11.3 \text{ cm}$$



Area (formula and work)

$$A = \frac{1}{2} bh$$

$$A \approx \frac{1}{2} (13.5 \text{ cm})(11.3 \text{ cm})$$

$$A \approx 76.3 \text{ cm}^2$$

TP =

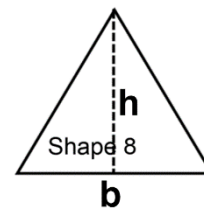
EP =

Shape 8 **Shape Name:** Triangle (Isosceles, non-right)

Dimensions (labeled picture and measurements)

$$b \approx 10.2 \text{ cm}$$

$$h \approx 8.5 \text{ cm}$$



Area (formula and work)

$$A = \frac{1}{2} bh$$

$$A \approx \frac{1}{2} (10.2 \text{ cm})(8.5 \text{ cm})$$

$$A \approx 43.4 \text{ cm}^2$$

TP =

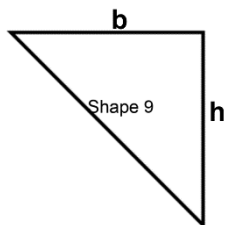
EP =

Shape 9 **Shape Name:** Right Triangle

Dimensions (labeled picture and measurements)

$$b \approx 13.4 \text{ cm}$$

$$h \approx 13.6 \text{ cm}$$



Area (formula and work)

$$A = \frac{1}{2} bh$$

$$A \approx \frac{1}{2} (13.4 \text{ cm})(13.6 \text{ cm})$$

$$A \approx 91.1 \text{ cm}^2$$

TP =

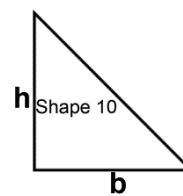
EP =

Shape 10 **Shape Name:** Right Triangle

Dimensions (labeled picture and measurements)

$$b \approx 10.2 \text{ cm}$$

$$h \approx 10.2 \text{ cm}$$



Area (formula and work)

$$A = \frac{1}{2} bh$$

$$A \approx \frac{1}{2} (10.2 \text{ cm})(10.2 \text{ cm})$$

$$A \approx 52.0 \text{ cm}^2$$

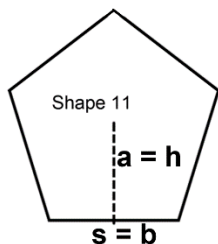
TP =

EP =

Shape 11 **Shape Name:** Pentagon

Dimensions (labeled picture and measurements)

$b \approx 8.7 \text{ cm}$
 $h \approx 6.5 \text{ cm}$



Area (formula and work)

$$A = n \frac{1}{2} bh = \frac{1}{2} nsa$$

$$A \approx 5 \frac{1}{2} (8.7 \text{ cm})(6.5 \text{ cm})$$

$$A \approx 141.4 \text{ cm}^2$$

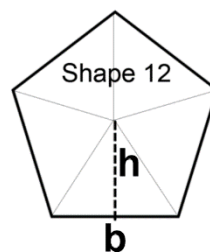
TP =

EP =

Shape 12 **Shape Name:** Pentagon

Dimensions (labeled picture and measurements)

$b \approx 6.2 \text{ cm}$
 $h \approx 4.7 \text{ cm}$



Area (formula and work)

$$A = n \frac{1}{2} bh$$

$$A \approx 5 \frac{1}{2} (6.2 \text{ cm})(4.7 \text{ cm})$$

$$A \approx 72.9 \text{ cm}^2$$

TP =

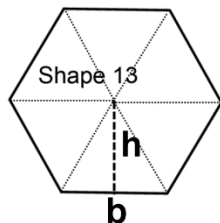
EP =

Shape 13 **Shape Name:** Hexagon

Dimensions (labeled picture and measurements)

$$b \approx 5.7 \text{ cm}$$

$$h \approx 5.0 \text{ cm}$$



Area (formula and work)

$$A = n \frac{1}{2} bh$$

$$A \approx 6 \frac{1}{2} (5.7 \text{ cm})(5.0 \text{ cm})$$

$$A \approx 85.5 \text{ cm}^2$$

TP =

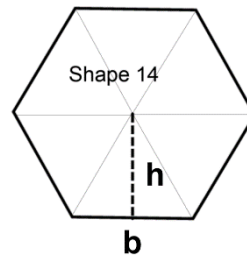
EP =

Shape 14 **Shape Name:** Hexagon

Dimensions (labeled picture and measurements)

$$b \approx 7.2 \text{ cm}$$

$$h \approx 6.2 \text{ cm}$$



Area (formula and work)

$$A = n \frac{1}{2} bh$$

$$A \approx 6 \frac{1}{2} (7.2 \text{ cm})(6.2 \text{ cm})$$

$$A \approx 133.9 \text{ cm}^2$$

TP =

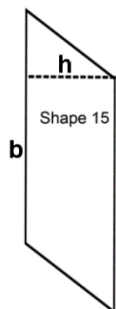
EP =

Shape 15 **Shape Name:** Parallelogram

Dimensions (labeled picture and measurements)

$$b \approx 18.0 \text{ cm}$$

$$h \approx 6.8 \text{ cm}$$



Area (formula and work)

$$A = bh$$

$$A \approx (18.0 \text{ cm})(6.8 \text{ cm})$$

$$A \approx 122.4 \text{ cm}^2$$

TP =

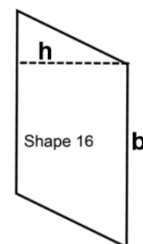
EP =

Shape 16 **Shape Name:** Parallelogram

Dimensions (labeled picture and measurements)

$$b \approx 14.2 \text{ cm}$$

$$h \approx 8.5 \text{ cm}$$



Area (formula and work)

$$A = bh$$

$$A \approx (14.2 \text{ cm})(8.5 \text{ cm})$$

$$A \approx 120.7 \text{ cm}^2$$

TP =

EP =

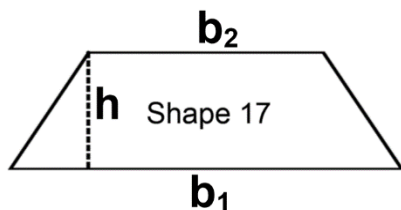
Shape 17 **Shape Name:** Trapezoid

Dimensions (labeled picture and measurements)

$$b_1 \approx 17.0 \text{ cm}$$

$$b_2 \approx 10.3 \text{ cm}$$

$$h \approx 5.0 \text{ cm}$$



Area (formula and work)

$$A = \frac{1}{2} h (b_1 + b_2)$$

$$A \approx \frac{1}{2} (5.0 \text{ cm})(17.0 \text{ cm} + 10.3 \text{ cm})$$

$$A \approx 68.3 \text{ cm}^2$$

TP =

EP =

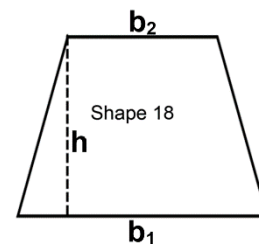
Shape 18 **Shape Name:** Trapezoid

Dimensions (labeled picture and measurements)

$$b_1 \approx 15.2 \text{ cm}$$

$$b_2 \approx 9.5 \text{ cm}$$

$$h \approx 11.3 \text{ cm}$$



Area (formula and work)

$$A = \frac{1}{2} h (b_1 + b_2)$$

$$A \approx \frac{1}{2} (11.3 \text{ cm})(15.2 \text{ cm} + 9.5 \text{ cm})$$

$$A \approx 139.6 \text{ cm}^2$$

TP =

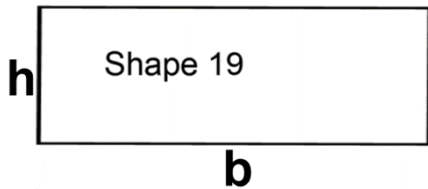
EP =

Shape 19 **Shape Name:** Rectangle

Dimensions (labeled picture and measurements)

$$b \approx 15.2 \text{ cm}$$

$$h \approx 5.2 \text{ cm}$$



Area (formula and work)

$$A = bh$$

$$A \approx (15.2 \text{ cm})(5.2 \text{ cm})$$

$$A \approx 79.0 \text{ cm}^2$$

TP =

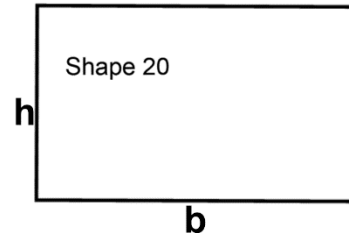
EP =

Shape 20 **Shape Name:** Rectangle

Dimensions (labeled picture and measurements)

$$b \approx 16.6 \text{ cm}$$

$$h \approx 10.0 \text{ cm}$$



Area (formula and work)

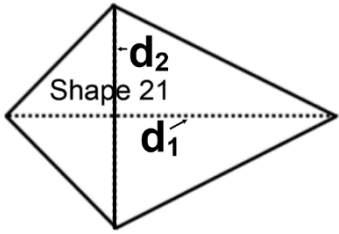
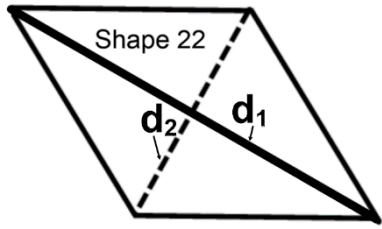
$$A = bh$$

$$A \approx (16.6 \text{ cm})(10.0 \text{ cm})$$

$$A \approx 166.0 \text{ cm}^2$$

TP =

EP =

<p>Shape 21 Shape Name: Kite</p> <p>Dimensions (labeled picture and measurements)</p> <p>$d_1 \approx 14.7$ cm $d_2 \approx 9.8$ cm</p>  <p>Area (formula and work)</p> <p>$A = \frac{1}{2} (d_1)(d_2)$</p> <p>$A \approx \frac{1}{2} (14.7 \text{ cm})(9.8 \text{ cm})$</p> <p>$A \approx 72.0 \text{ cm}^2$</p> <p>TP =</p> <p>EP =</p>	<p>Shape 22 Shape Name: Rhombus</p> <p>Dimensions (labeled picture and measurements)</p> <p>$d_1 \approx 20.5$ cm $d_2 \approx 11.4$ cm</p>  <p>Area (formula and work)</p> <p>$A = \frac{1}{2} (d_1)(d_2)$</p> <p>$A \approx \frac{1}{2} (20.5 \text{ cm})(11.4 \text{ cm})$</p> <p>$A \approx 116.9 \text{ cm}^2$</p> <p>TP =</p> <p>EP =</p>
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Suggestions

- The point of this Geometric Probability lesson is for students to discover that the probability of a dart hitting any of the shapes is the same as the ratio of each area to the total dartboard area.
- First, have students select their shapes and trace them on the dartboard. It is helpful if in a class ALL students select one of their shapes to be the same, for instance #1.
- Second, have them take a picture of the dartboard and create their GeoGebra file. They should follow the directions on the GeoGebra HELPS sheet as they create a random point generator. Note that GeoGebra updates the look of their website, but the Geometry Classic often looks similar. So, even if the HELPS sheet is not exactly how the current GeoGebra site looks like, the steps should still be similar.
- Third, have students find EP. After all students have created a dartboard and a GeoGebra file, have them all calculate the experimental probability for their similar shape (i.e., Shape #1). For students who may not have yet finished their GeoGebra file, send them a link to a finished GeoGebra file so that they can all participate in this step together. Students may use this link to access a GeoGebra Dartboard with a random point generator: <https://www.geogebra.org/classic/K67t4uFJ> Each student should count the number of points in the similar shape (i.e., Shape #1). The teacher can create a Google Sheet with all the students' numbers. Then, the teacher shows the students how to type in a formula to average the numbers. The average EP (experimental probability) should be very close to the TP (theoretical probability). Have a class discussion about how important it is to have many trials (experiments) rather than taking just one trial (experiment) to determine a probability that is close to the theoretical probability.

- Here is a sample of the Google Sheets with some suggestions:
<https://docs.google.com/spreadsheets/d/1c5thWakzr0x2EbCXbWb8vtFlhA4QsijGAFv4y48zHPw/edit?usp=sharing>
 I do not share this file with students. I want each student to have the opportunity to create his/her own spreadsheet.
- The measurements in the solutions are rounded to the tenths place. This creates great discussions in the classroom on accuracy when rounding is done within a problem. Note that dimension measurements of the shapes may vary if the format of the SHAPES pages is changed.
- This project can be modified for all levels of students and classes. The teacher has a lot of options with this project; discretion can be made in determining what parts of the project would be best for student learning.
 - The teacher can guide students through the project when more help is needed for the students.
 - The modified 45 point rubric can be used for students who would not yet be ready to create a GeoGebra file.
 - The entire project can be assigned and students can work in class for a period and then do the rest on their own.
 - The entire project can be done in the classroom.
 - The teacher could use just part of the lesson plan and teach the students TP and EP in even 30 minutes!
 - Teachers who want to allow students to create and make it more of a STEAM project can have students create a theme to the dartboard. Students could trace the shapes on colored paper, cut out the colored shapes, and glue them onto a colored dartboard. Students could draw on the shapes and color a design.
- Lesson Extension: Use Geometric Probability as a tool to find the area of non-polygonal shapes.
 - Trace your hand on an 11 inch by 17 inch sheet of paper.
 - Follow the GeoGebra programming steps as you did for your shapes lesson.
 - Calculate the Experimental Probability (EP) of your hand.
 - Use a proportion to solve for the area of your hand. Set the EP fraction equal to the ratio of your unknown hand area to the known area of the dartboard.

Acknowledgements

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