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for Innovation in Math Teaching

Searching for Intersections

Solving Linear Equations by Graphing

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Table of Contents

Lesson Goals

Solving equations using graphing techniques is applicable in real-world problems for its visual explanation of a solution. Graphing has been introduced, particularly using equations in slope-intercept form. Approaching a mathematical concept as a tool to transcend the classroom and applying it to a real-world situation is extremely important as they build their reasoning skills. This lesson allows students to discover why graphing can show one, zero, or infinite solutions of two equations, and they ultimately will use their understanding to find the location of a prize on a map.

This lesson can be used as an initial discovery activity and can continue over multiple days into an extension activity.

Student Outcomes

Students will use existing graphs to discover that the same solutions can exist between multiple equations. They will find that these solutions are found by graphing intersecting (or parallel or overlapping) lines. By the end of the lesson, they will be able to use their understanding of solving equations by graphing to locate a secret location on a map, where a prize will be located.

Common Core Practice Standards

CCSS.MATH.CONTENT.8.EE.C.8.A
Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Prerequisite Knowledge

Students should have exposure and basic understanding of the following vocabulary terms: linear equation, slope, y-intercept, perpendicular, bisector, midpoint, and opposite reciprocal. They should also have had lessons immediately prior that explore graphing lines in slope intercept form, midpoint formula, as well as basic coverage of parallel and perpendicular lines.

Time Required

Preparation Time: 30-60 minutes Class Time: 1 hour 20 minutes

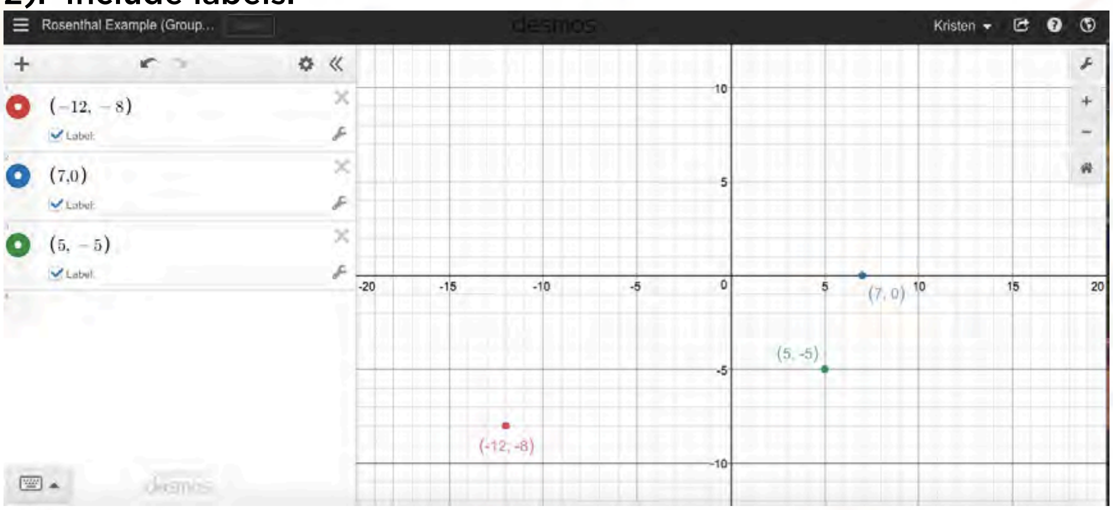
Materials

- Prize (candy, gum, homework pass, etc.)
- Paper/plastic bags to put prize inside. Tape a sign or write the corresponding map number on the outside of the bag.
- Straightedge if students desire to draw their lines neatly in the Scavenger Hunt.
- Protractors, if students desire to check that they have found a perpendicular bisector in the Scavenger Hunt.
- Solving Equations by Graphing Handout
- Prize maps (10 different versions)
- Prize map answer keys (10 different versions)
- For teacher, computer and internet access to

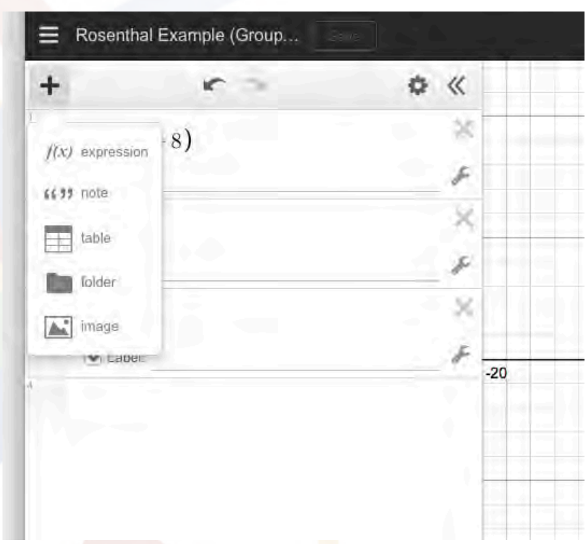
Set Up

Before class, choose how many different versions of the prize maps you want to use. Use the points from the given handouts (see page 13). An easy way to overlay the graph is to insert your desired image(s) into Desmos online graphing calculator (some points will be estimated in order to remain integers). If you prefer to use your own picture, you can also plot any of the point configurations in Desmos and insert your own image behind the points. See below for some guidance on Desmos images.

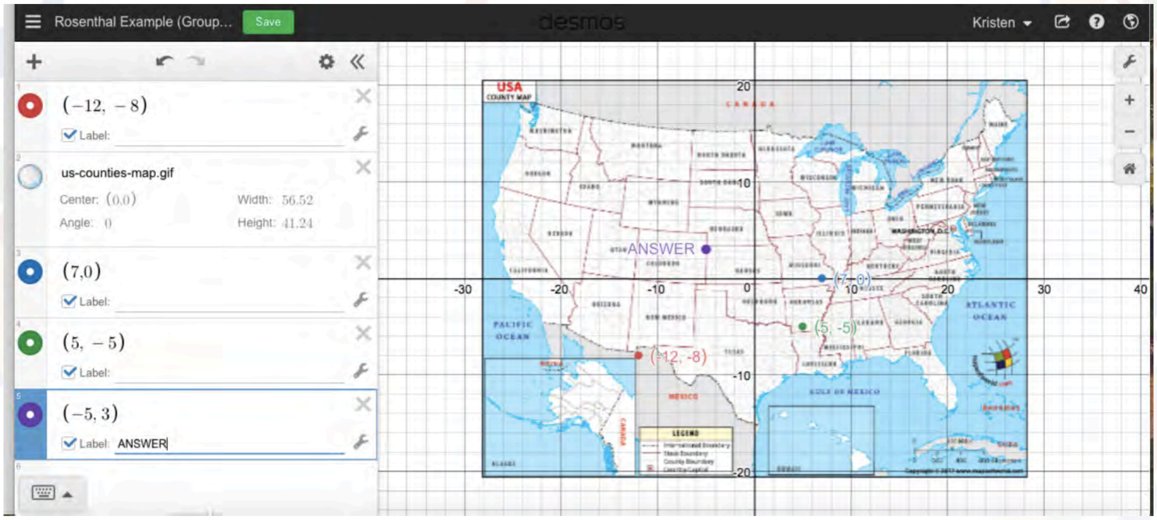
First plot the point configuration you would like to use (this is Group 2). Include labels.



Second, choose the “plus sign” drop down menu and select “image”. You will then be able to choose the picture you’d like to insert from your saved files.



Finally, once the image is inserted you will both see the points you already made, add new points (such as the answer shown here, although you will not want this on the student-version), and resize the image.



One complete Desmos example with all teacher-made lines is attached, and the other nine point configurations in Desmos are also attached. You can create up to ten different prize locations based on the different point configurations. You can then choose whether you want to place a prize at the location, or reward the students in the classroom once they have found it on the map. Note that that prize location is marked on each version of the handout, so you will not want to mark it on the handout given to the students.

If you choose to place prizes at the locations on the Prize Map, mark each of the bags with the corresponding map number so that students will be able to identify that they have found the correct location.

Make enough copies of the Solving Equations by Graphing handout and the Prize Maps for each student.

Make sure each student has a straightedge and protractor, should they choose to use them for the Scavenger Hunt.

Introduction to the Activity [5 minutes]

Begin the lesson by asking students to describe what a solution of a linear equation looks like. Have them “think, pair, and share” with a neighbor, noting how closely their answers match. If students are not landing on the answer, prompt them by asking if the following is a solution of the equation: *“Is (3, 4) a solution of y = 2x – 1?’* Allow students time to work through the example while you circulate to ensure that students are figuring out that they need to evaluate the linear equation for the given (x, y) coordinate.

Discovery [35 minutes]

Students will discover the relationship between solving equations and graphing by working through a tiered discovery activity. This can be a teacher-led or independent activity based on the ability levels of your students.

Using the questioning guide on page 11 to work with students through the Solving Equations by Graphing Handout. Remind students as needed of previous graphing concepts and how they are also being used in this new context.

Answer Key is also provided in the handouts section.

Exploration [35 minutes]

Students will apply their knowledge from the discovery to find a secret location. They should have prerequisite knowledge of finding midpoints and slopes, and applying opposite reciprocals as perpendicular slopes. If a short review is needed of these concepts, insert here as necessary.

Split students into groups of two and hand out copies of the prize map with marked points to each group (see Handouts section). Each partner may have a copy, or just one can be handed out per group. There are ten different versions, each leading to a different location, so each group can work on a different version, or you may limit them to a single location.

Tell students that their goal is to find a location on the map that lies on the intersection of the three perpendicular bisectors of the sides of the triangle made from the points (which is itself a single point), where a prize will be located. Depending on the level of the class, you may leave that as the only direction, or guide students to use the points to first find a midpoint of a side, then to find the slope of the same side, and finally to find the perpendicular slope by getting the opposite (negative) reciprocal of the original slope of the side. Then, these students should draw the perpendicular bisectors of each line and find the coordinates of their intersection point. Keep in mind that the goal is graphing lines to find their intersections, so students should not need to get into finding equations. A sample of a student's process is shown below:

The students will find the midpoint of each side to find the coordinates of each bisector, determine the slope of each side, and take the opposite reciprocal of the slopes to calculate the perpendicular slopes.

Sample Student Work (Exporation Activity)
Group 1

→ Line AB (Red, (-12,-8) and (-5,3))

$$\text{midpoint}_{AB} = \left(\frac{-12-5}{2}, \frac{-8+3}{2}\right) = \left(\frac{-17}{2}, \frac{-5}{2}\right) = \boxed{(-8.5, -2.5)}$$

$$\text{slope}_{AB} = \frac{3-(-8)}{-5-(-12)} = \frac{3+8}{-5+12} = \frac{11}{7}$$

$$\boxed{\text{slope}_{\perp AB} = \frac{-7}{11}}$$

→ Line BC (Blue, (-5,3) and (5,-5))

$$\text{midpoint}_{BC} = \left(\frac{-5+5}{2}, \frac{3-5}{2}\right) = \left(\frac{0}{2}, \frac{-2}{2}\right) = \boxed{(0,-1)}$$

$$\text{slope}_{BC} = \frac{-5-3}{5-(-5)} = \frac{-5-3}{5+5} = \frac{-8}{10} = \frac{-4}{5}$$

$$\boxed{\text{slope}_{\perp BC} = \frac{5}{4}}$$

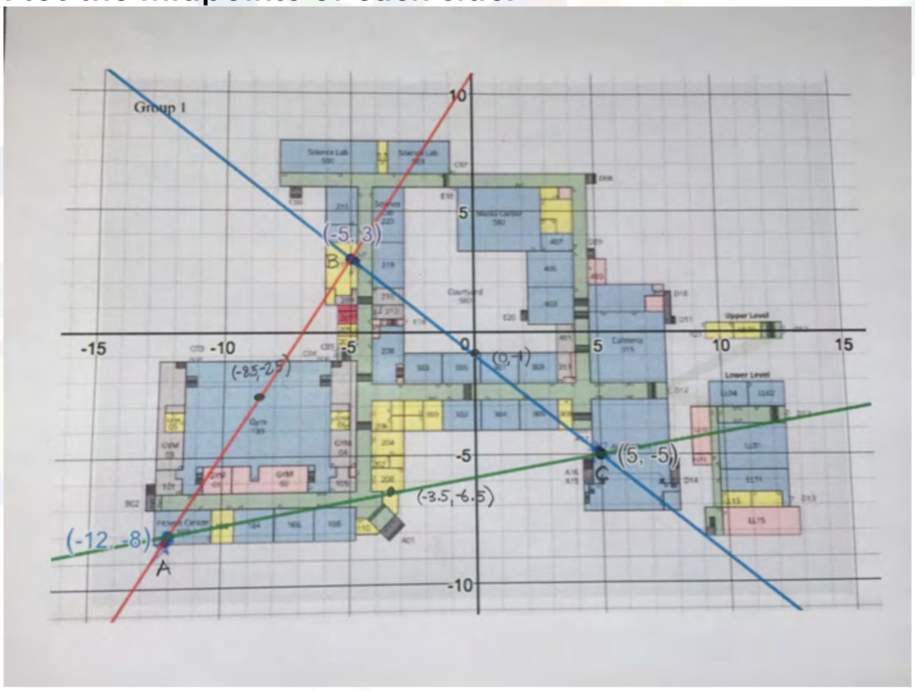
→ Line AC (Green, (-12,-8) and (5,-5))

$$\text{midpoint}_{AC} = \left(\frac{-12+5}{2}, \frac{-8-5}{2}\right) = \left(\frac{-7}{2}, \frac{-13}{2}\right) = \boxed{(-3.5, -6.5)}$$

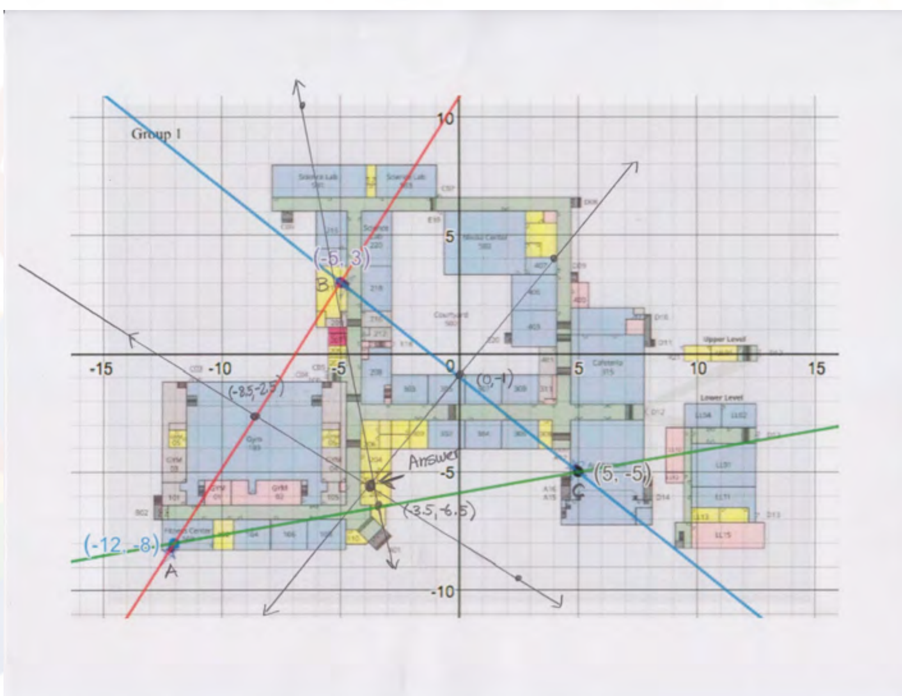
$$\text{slope}_{AC} = \frac{-5-(-8)}{5-(-12)} = \frac{-5+8}{5+12} = \frac{3}{17}$$

$$\boxed{\text{slope}_{\perp AC} = -\frac{17}{3}}$$

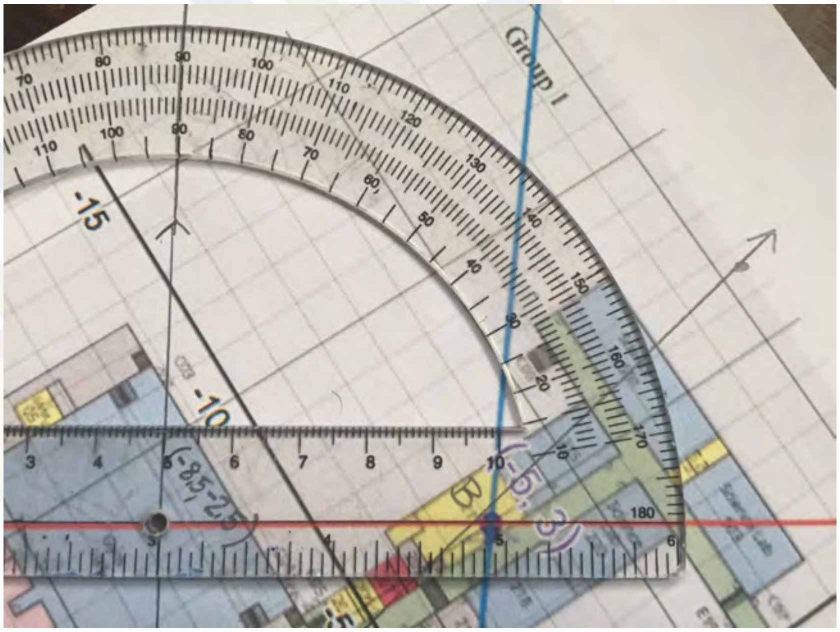
Plot the midpoints of each side:



Determine a second point of each perpendicular bisector, then draw the lines through each set of points. The intersection of the perpendicular bisectors if the location of the prize.



If desired, the student may check with a protractor to see that their bisector line is perpendicular to the side of the triangle.



Let students work in groups to discuss their strategy, and allow them to check their location with you before they go out of the classroom to confirm their location. It is the teacher's decision to include a prize to look for or not, as well as whether the students are allowed to leave the classroom before they have the correct answer.

Summarizing [5 minutes]

After students have returned from searching for the prizes, gather the class back together to summarize the understanding gained from the lesson. Some suggested questions are:

What was the most important piece of information you used to help you locate the prizes?

Why could graphing to find solutions of linear equations be useful in real-world thinking?

What information from today might you need to apply in order to figure out

Essential Questions

How are solutions of linear equations able to be seen on a coordinate plane?

Why can pairs of linear equations have different amounts of solutions?

Questioning Guide for Solving Equations by Graphing Handout:

Question for #1, #2: What different (x, y) coordinates lie on the graphs?

Question for #3: Do the lines pass over the same point anywhere on the graphs?

Question for #4, #5: What is the slope and y-intercept of each graph?

Question for #8: Do lines always continue to travel in the same direction, or can they curve around again?

Questions for #11: How did you decide where to start your graphs? Did you have the same lines as your neighbor?

Question for #12: If there are no solutions, can the graphs ever intersect?

Question for #13: What type of lines are created when the equation has the same slope, but different y-intercepts?

Questions for #14: What does overlapping mean? Will lines with infinite solutions always overlap?

Question for #15: What kind of lines make 90 degree angles, and what have we learned about slopes that would confirm this?

Questioning Guide for Exploration:

Question: What does a perpendicular bisector do to a line?

Question: How will we find the point where the perpendicular line should pass through a side?

Question: Why do we not actually need to use an equation of a line to find the intersection of the perpendicular bisectors?

Adaptations and Extensions

Intentionally pair up students of stronger and lesser understanding during the Discovery and Exploration.

Give a short review of previous concepts needed to complete the Exploration Scavenger Hunt.

Create a pre-lesson video to introduce/review vocabulary necessary for the activity.

Students work online on GeoGebra to create online versions of the Exploration section.

Use the Exploration as a pre-lesson introduction activity to introduce solutions made by intersecting lines.

Use the Exploration as an extension project to assess understanding of several concepts at once, including midpoints, perpendicular slopes, and finding equations.

Use any map of your choosing and transfer the points from the handout onto your own map using Desmos to overlay the graph.

For advanced classes, also include finding the equations of the lines in the Exploration.

If time is short for the Exploration, either split the class into groups and assign each group one of the sides to find the perpendicular bisector for or assign one side to each member in a small group.

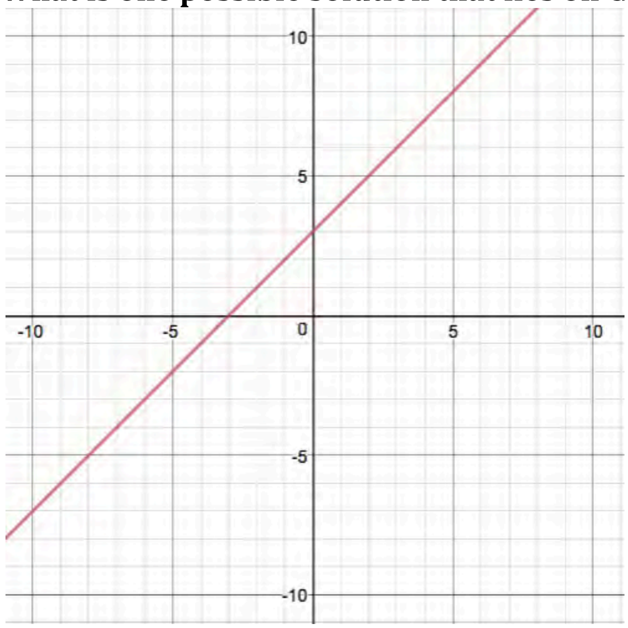
For Future Study

Circumcenters provide a valuable foundation for real-world applications. Students can use circumcenters as a springboard in evaluating city planning problems, learning to be critical of the given situation, and using mathematical processes to evaluate the potential outcomes. The Exploration guides students towards the concept of the point of concurrency of the perpendicular bisectors, which is known as the circumcenter. In particular, the circumcenter has several unique properties, including its equidistance from the three vertices of the given triangle.

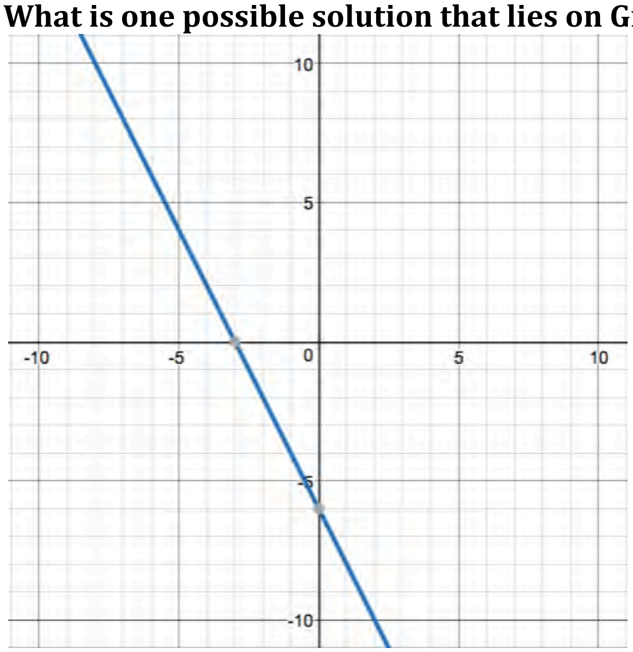
Handouts Section

Solving Equations by Graphing Handout Name: ______________________________

1. **What is one possible solution that lies on Graph 1?** ________________



2. **What is one possible solution that lies on Graph 2?** ________________



3. **Is there a point that is a solution to both graphs? If so, what is it?**

____

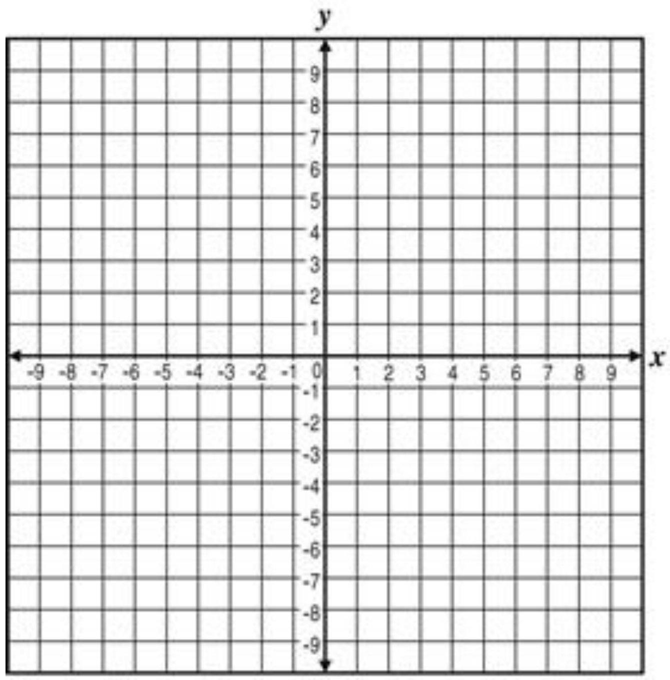
4. **What is the equation for Graph 1's line? (Remember y = mx + b!)**

____

5. **What is the equation for Graph 2's line?**

____

6. **Graph both lines on the coordinate plane below:**

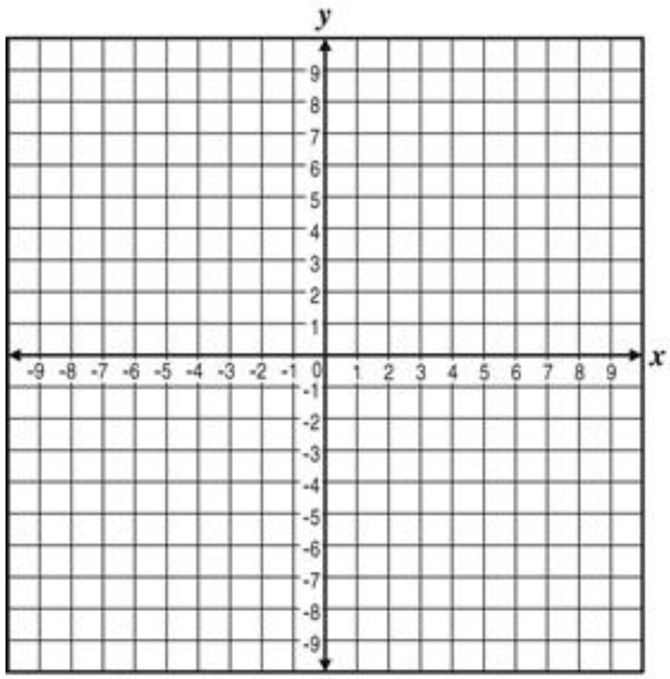


7. **What do you notice about the lines you graphed in #6 and the solution you came up with in #3?**

8. **How do you know that this is the only solution?**

9. **Do you think graphs with other amounts of solutions exist? Why?**

10. Sketch two lines below that have a solution at (3, 0). Write their equations in the spaces provided.



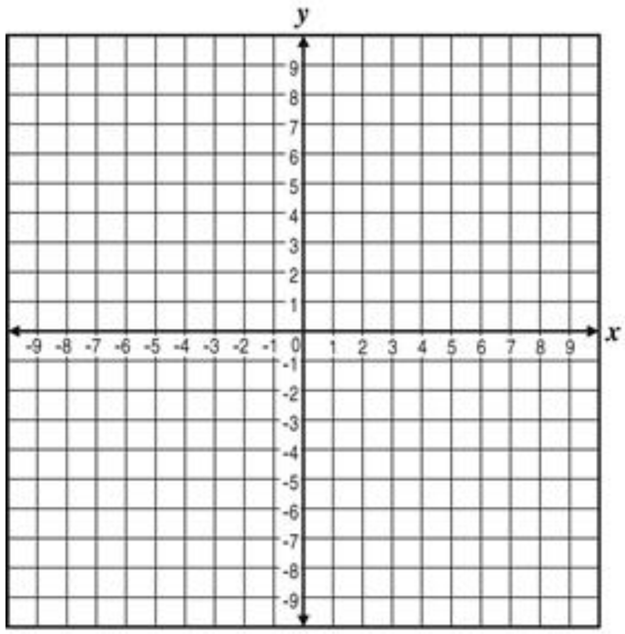
Equation 1: ________________________________

Equation 2: ________________________________

11. Describe the decisions you made when you sketched your lines.

One more page…you can do it! →

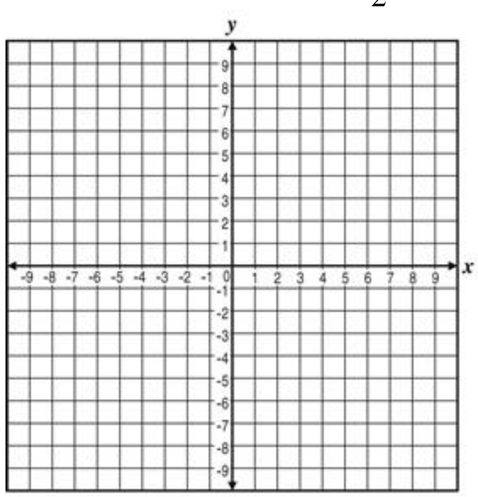
12. **What do you think lines with no solutions look like? Sketch them below.**



13. **What do the slopes of these lines have in common?**

14. **How many solution(s) do overlapping lines have in common?**

15. **Last question! What do you notice is special about these intersecting lines? Graph them!** $y = \frac{1}{2}x + 1$ **and** $y = -2x + 3$

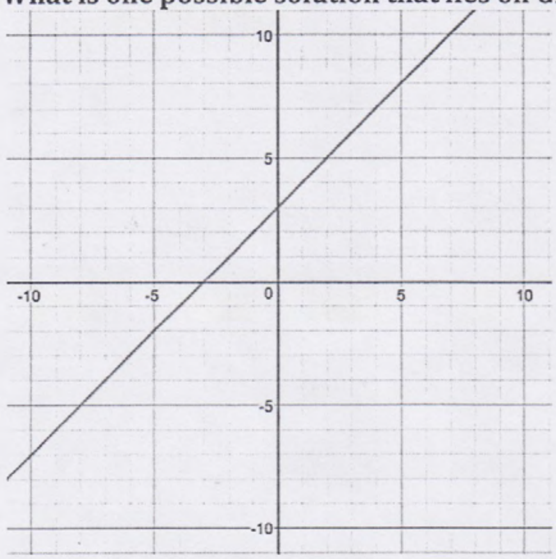


Do the slopes have anything to do with this? Hmm…

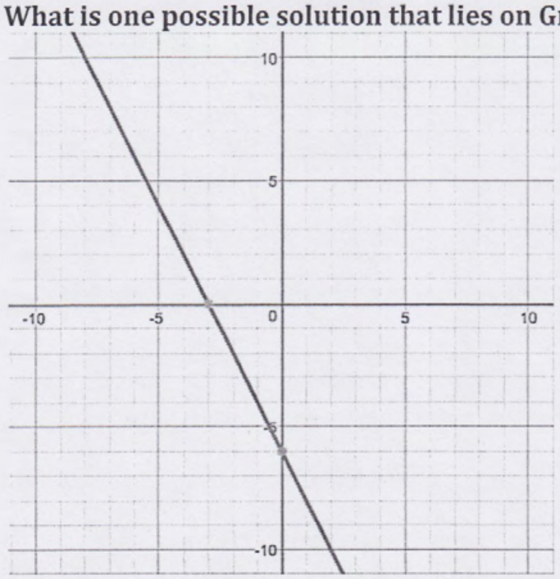
Solving Equations by Graphing Handout Name: Answer Key

(Answers may vary = Amv)

1. **What is one possible solution that lies on Graph 1?** (2, 5) (Amv)



2. **What is one possible solution that lies on Graph 2?** (0, -6) (Amv)



3. **Is there a point that is a solution to both graphs? If so, what is it?**

(-3,0)

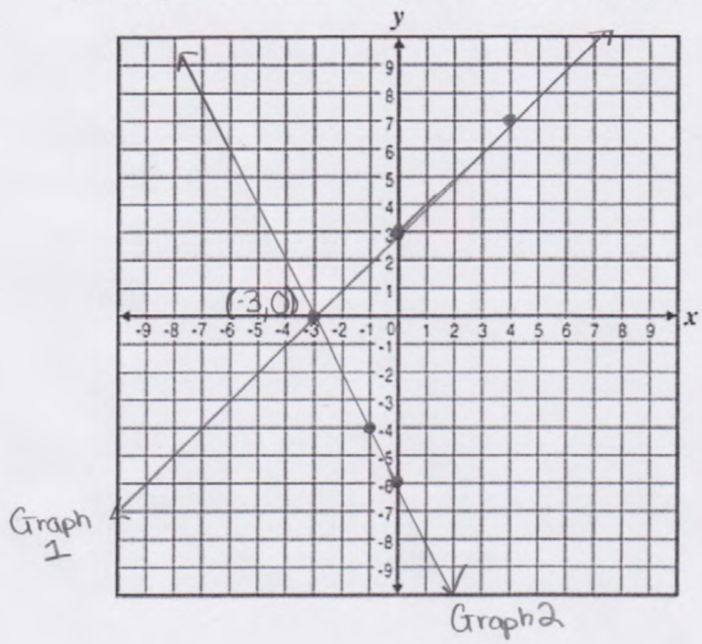
4. **What is the equation for Graph 1's line? (Remember y = mx + b!)**

   $y = x + 3$

5. **What is the equation for Graph 2's line?**

   $y = -2x - 6$

6. **Graph both lines on the coordinate plane below:**



7. **What do you notice about the lines you graphed in #6 and the solution you came up with in #3?**

   The point of intersection is the same as #3's answer.

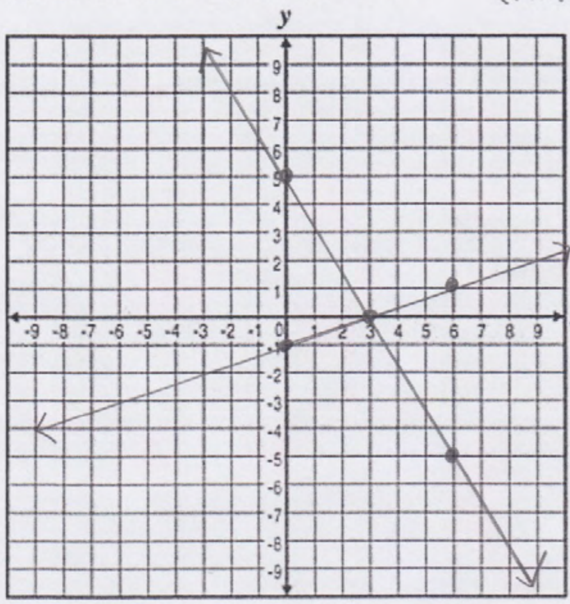
8. **How do you know that this is the only solution?** (Amv)

   Lines by definition continue infinitely in opposite directions, so they cannot "curve" back around to intersect a second time.

9. **Do you think graphs with other amounts of solutions exist? Why?** (Amv)

   Yes, because linear equations may also either overlap or never intersect.

10. **Sketch two lines below that have a solution at (3, 0). Write their equations in the spaces provided.** (Amv)



Equation 1: $y = \frac{1}{3}x - 1$

Equation 2: $y = -\frac{5}{3}x + 5$

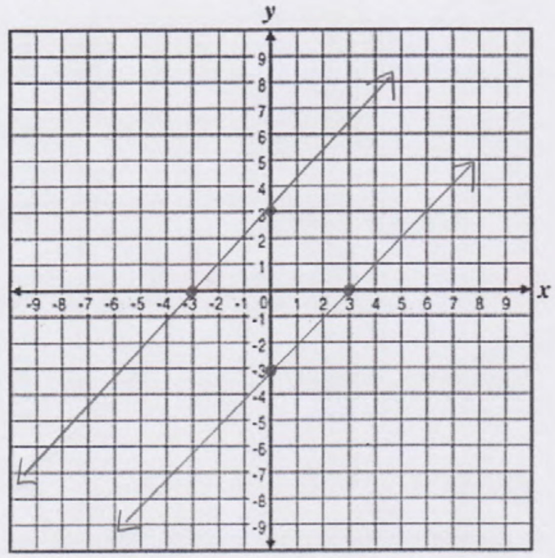
11. **Describe the decisions you made when you sketched your lines.** (Amv)

We know the lines must intersect at (3,0), so I started by plotting (3,0) on the coordinate plane. I chose a second point for equation 1, then drew the line between them (in this case, the second point was the y-intercept). I did the same for equation 2.

One more page...you can do it! →

12. **What do you think lines with no solutions look like? Sketch them below.**

(Amu)



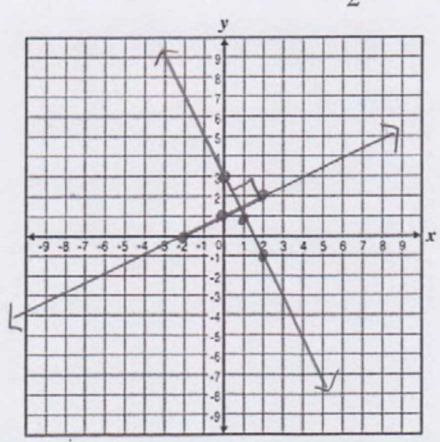
13. **What do the slopes of these lines have in common?**

They have the same slope.

14. **How many solution(s) do overlapping lines have in common?**

Infinite solutions

15. **Last question! What do you notice is special about these intersecting lines? Graph them!** $y = \frac{1}{2}x + 1$ **and** $y = -2x + 3$



They make 90° angles when intersecting, making the lines perpendicular.

Do the slopes have anything to do with this? Hmm...

The slopes are opposite recipracals.


Group 1
(-5, 3)
(5, -5)
(-12, -8)
-15
-10
-5
0
5
10
15
Science Lab
Media Center
Courtyard
Cafeteria
Gym
Fitness Center
Upper Level
Lower Level

Group 2

Science Lab 501
Science Lab 503
Media Center 502
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Upper Level
Lower Level
(7, 0)
(5, -5)
(-12, -8)
-15
-10
-5
0
5
10
15

Group 3
(-3, 8)
(3, -4)
(-18, -6)
Science Lab 501
Science Lab 503
Media Center 502
Courtyard 500
Gym 103
Fitness Center 100
Auditorium 310
Upper Level
Lower Level
10
5
0
-5
-10
-15
20
15

Group 4

10
5
-5
-10
-15
15
0
(0, 7)
(8, -4)
(-9, -8)
Science Lab
501
Science Lab
503
Media Center
502
Science
Lab
220
Courtyard
500
Cafeteria
315
Auditorium
310
Gym
103
Fitness Center
100
Upper Level
Lower Level
UL01
LL04
LL02
LL01
LL10
LL11
LL12
LL13
LL15
GYM
03
GYM
04
GYM
01
GYM
02
215
213
211
209
207
205
203
216
212
218
208
303
307
309
311
302
304
306
308
300
206
204
202
200
407
405
403
401
400
101
105
106
108
110
D08
D09
D10
D11
D12
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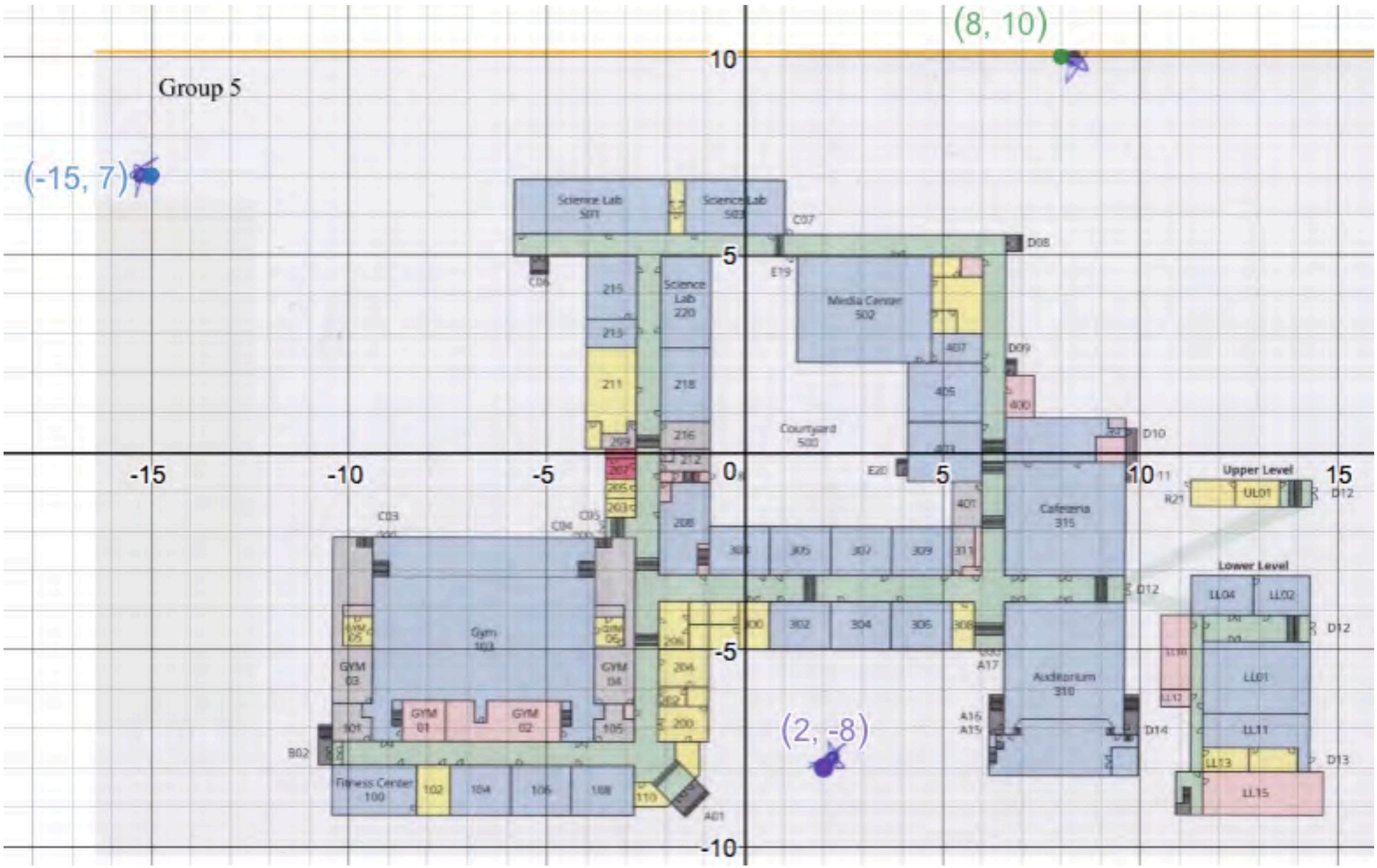
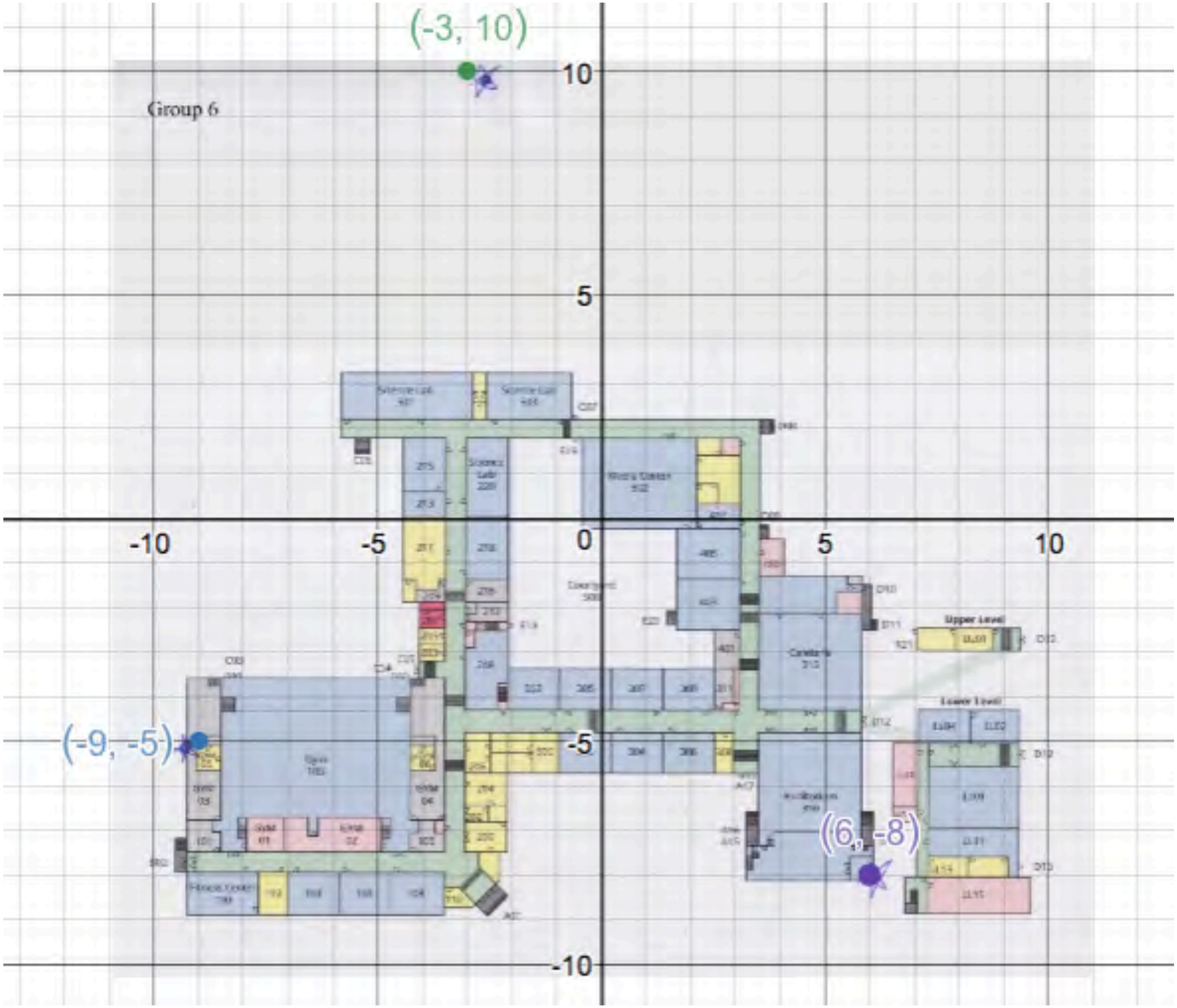
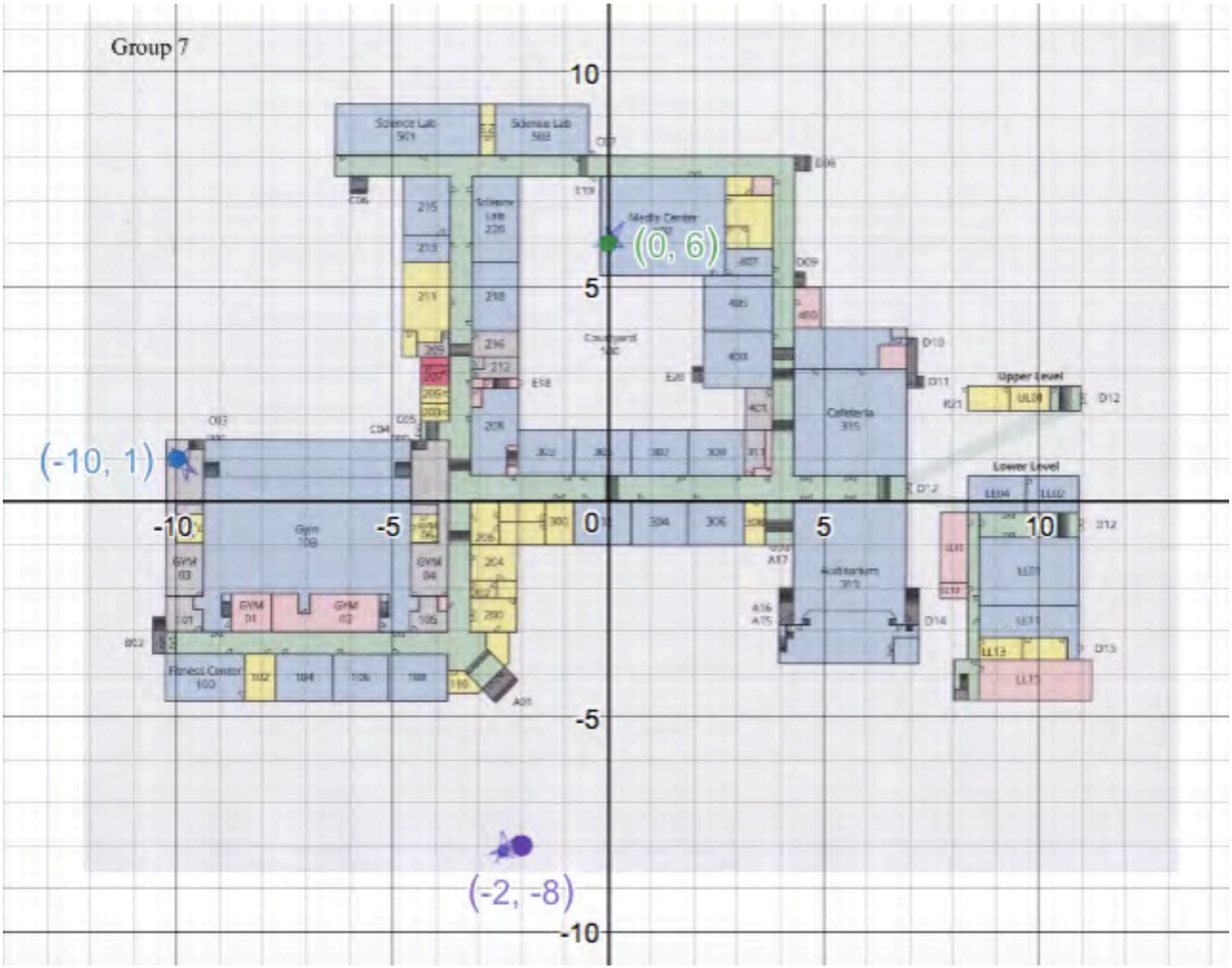
C05
C06
B02
A01
A15
A16
A17
R21


Group 5
(8, 10)
(-15, 7)
(2, -8)
Science Lab 501
Science Lab 503
Science Lab 220
Media Center 502
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Fitness Center 100
Upper Level
Lower Level
UL01
LL01
LL02
LL04
LL11
LL13
LL15
GYM 01
GYM 02
GYM 03
GYM 04
215
213
211
218
216
212
208
207
205
203
209
206
204
202
200
300
302
304
306
308
303
305
307
309
311
400
401
403
405
407
102
104
106
108
110
105
101
C03
C04
C05
C06
C07
D08
D09
D10
D12
D13
D14
E19
E20
A01
A15
A16
A17
B02
R21
-15
-10
-5
0
5
10
15


Group 6
(-3, 10)
(-9, -5)
(6, -8)
Upper Level
Lower Level



Group 7
Science Lab
501
Science Lab
503
Media Center
Cafeteria
315
Auditorium
Gym
108
GYM
03
GYM
04
GYM
01
GYM
02
Fitness Center
100
Upper Level
Lower Level
(0, 6)
(-10, 1)
(-2, -8)
10
5
-5
-10
0

Group 8

(5, 8)
(-5, 0)
(6, -10)
Science Lab 501
Science Lab 503
Media Center 502
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Fitness Center 100
Upper Level
Lower


Group 9
10
5
-5
-10
-10
-5
0
5
10
(-10, -1)
(-2, 0)
(-3, -8)
Upper Level
Lower Level

Group 10


(-8, 9)
(12, 8)
(-6, 0)
-15
-10
-5
0
5
10
15
Science Lab 501
Science Lab 503
C07
D08
C06
E19
215
Science Lab 220
Media Center 502
213
407
D09
211
218
405
400
Courtyard 500
216
212
403
D10
209
207
E18
E20
D11
Upper Level
UL01
R21
D12
C03
C04
208
401
Cafeteria 315
303
305
307
309
311
Lower Level
LL04
LL02
Gym 103
300
302
304
306
308
LL10
LL01
LL12
GYM 05
GYM 06
206
204
A17
Auditorium 310
GYM 03
GYM 04
202
GYM 01
GYM 02
200
A16
A15
D14
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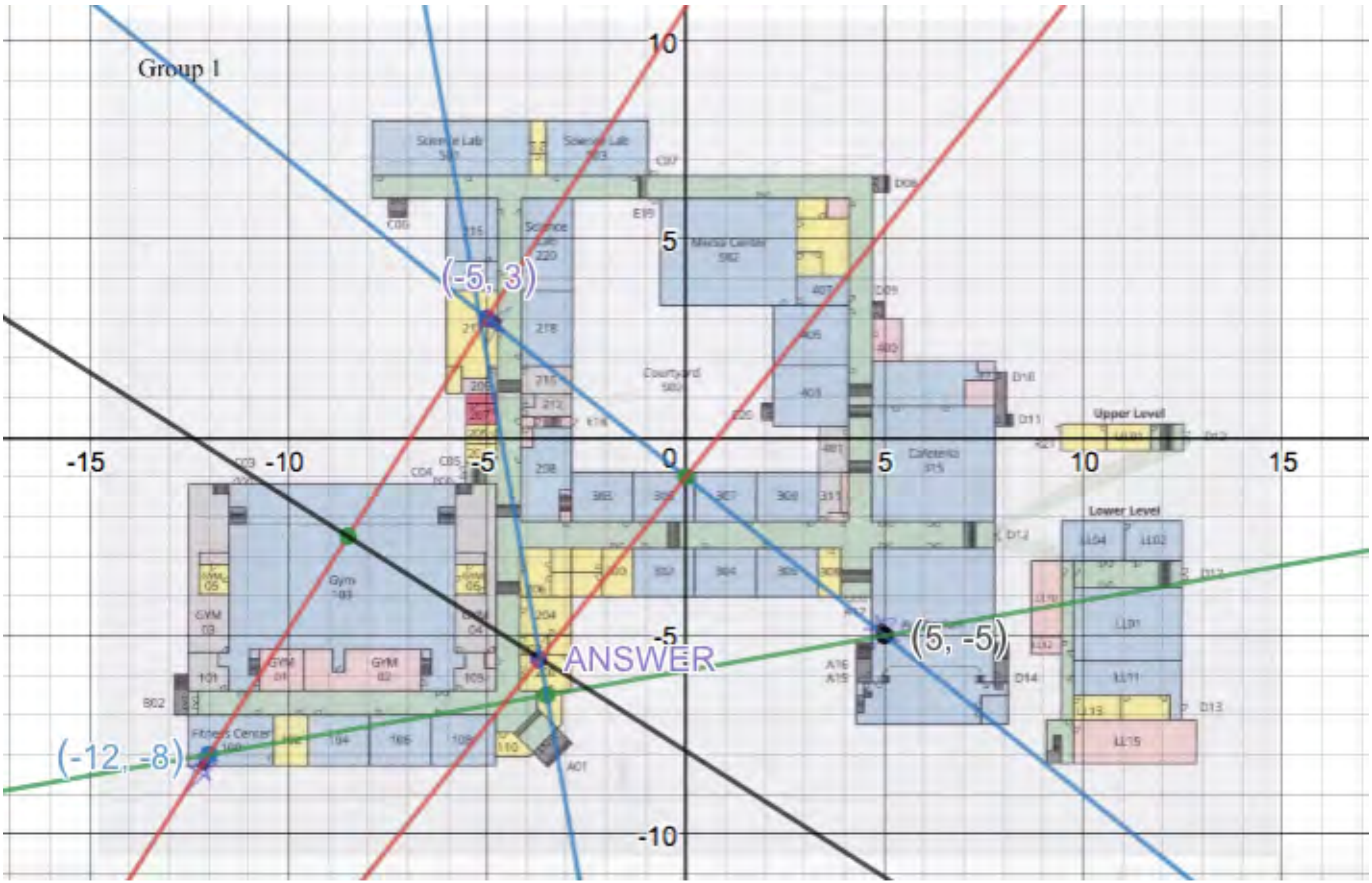
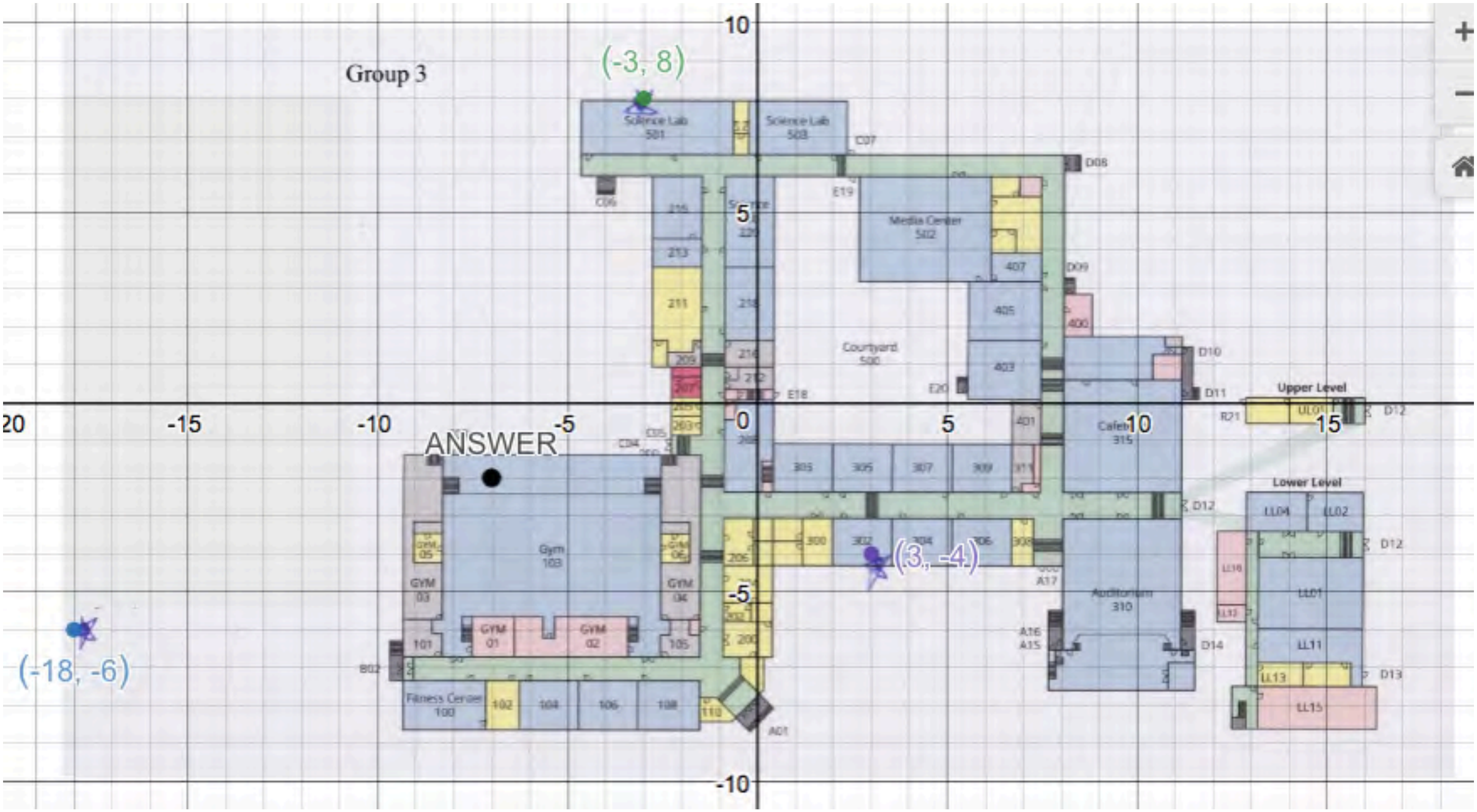
LL13
D13
B02
Fitness Center 100
102
104
106
108
110
A01
LL15



Group 1
(-5, 3)
(5, -5)
(-12, -8)
ANSWER
Science Lab
Media Center
Courtyard
Cafeteria
Gym
Fitness Center
Upper Level
Lower Level
10
5
0
-5
-10
-15
15

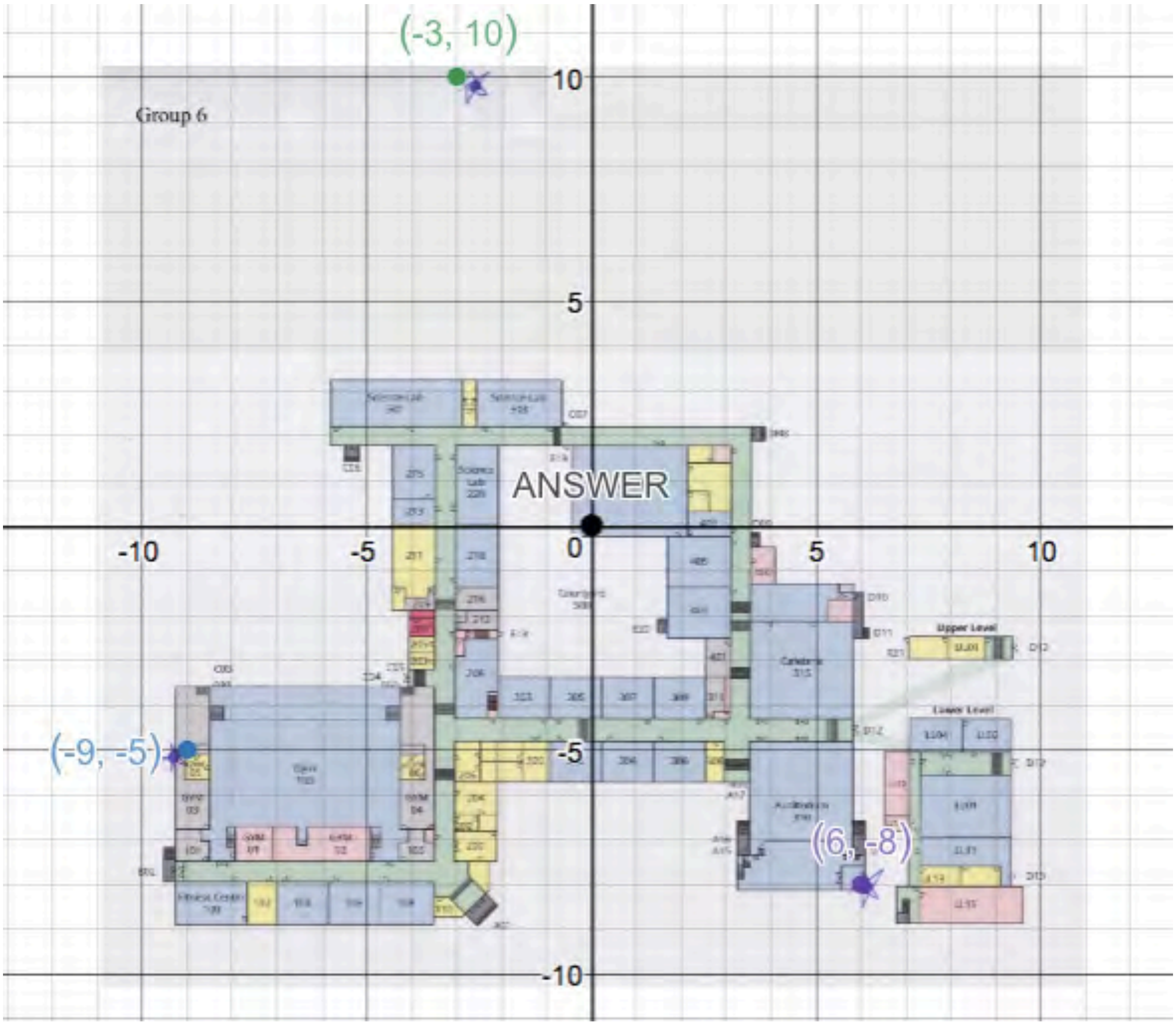

Group 2
ANSWER
(7, 0)
(5, -5)
(-12, -8)
Science Lab 501
Science Lab 503
Media Center 502
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Upper Level
Lower Level
-15
-10
-5
0
5
10
15


Group 3
(-3, 8)
(3, -4)
(-18, -6)
ANSWER
Science Lab 501
Science Lab 503
Media Center 502
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Fitness Center 100
Upper Level
Lower Level
LL04
LL02
LL01
LL11
LL13
LL15

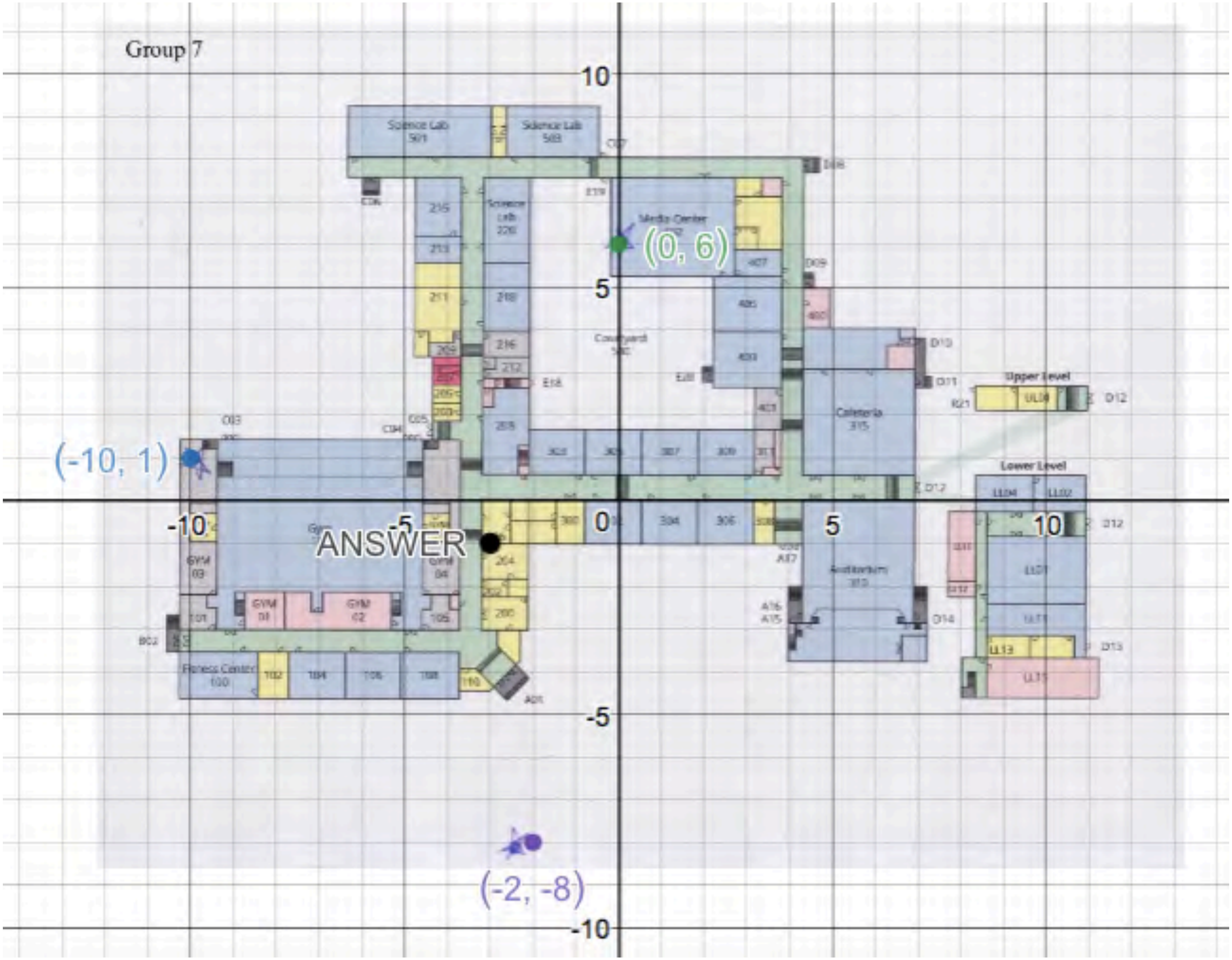
Group 4

(0, 7)
(8, -4)
(-9, -8)
ANSWER
Science Lab 501
Science Lab 503
Media Center 502
Science Lab 220
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Fitness Center 100
Upper Level
Lower Level
UL01
LL01
LL02
LL04
LL10
LL11
LL12
LL13
LL15
GYM 01
GYM 02
GYM 03
GYM 04
GYM 05
GYM 06
101
105
106
108
110
200
202
204
206
208
211
212
213
215
216
218
300
303
306
307
309
311
401
403
405
407
490
A01
A15
A16
A17
B02
C03
C04
C05
C06
D08
D09
D10
D11
D12
D13
D14
E18
E19
E20
R21
10
5
0
-5
-10
-15
15


Group 5
(8, 10)
(-15, 7)
ANSWER
(2, -8)
Science Lab 501
Science Lab 503
Science Lab 220
Media Center 502
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Fitness Center 100
Upper Level
Lower Level
UL01
LL01
LL02
LL04
LL11
LL13
LL15
GYM 01
GYM 02
GYM 03
GYM 04
211
213
216
218
208
204
200
102
104
106
108
110
302
304
306
305
307
309
401
405
407
400
D08
D09
D10
D12
D13
D14
C07
C03
A01
A15
A16
A17
B02
E19
E20
R21

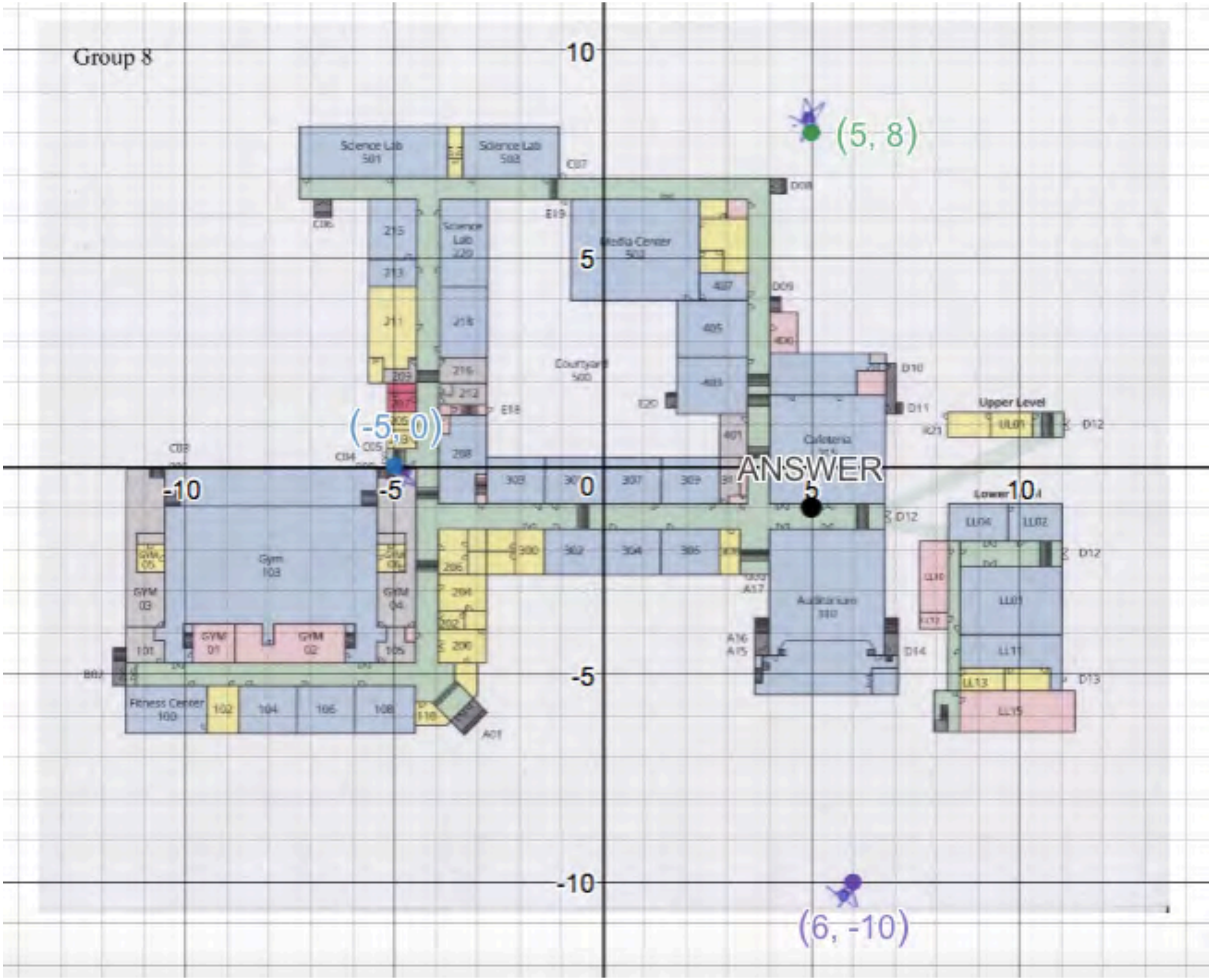

Group 6
(-3, 10)
(-9, -5)
(6, -8)
ANSWER
Upper Level
Lower Level
10
5
0
-5
-10

Group 7

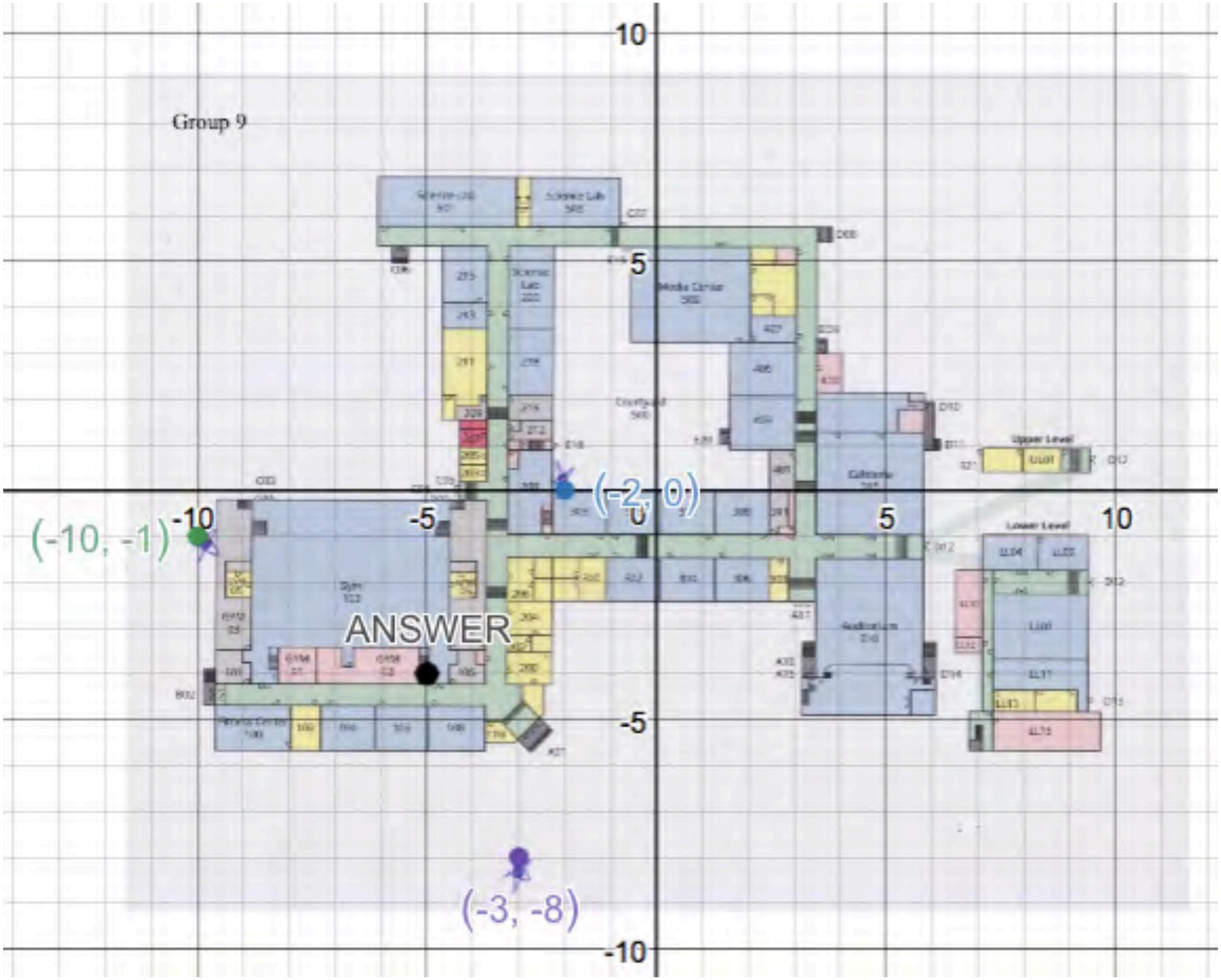


10
5
0
-5
-10
(0, 6)
(-10, 1)
(-2, -8)
ANSWER
Science Lab 501
Science Lab 503
Media Center
Courtyard
Cafeteria 315
Auditorium 313
Upper Level
Lower Level
Gym
GYM 01
GYM 02
GYM 03
Fitness Center 100

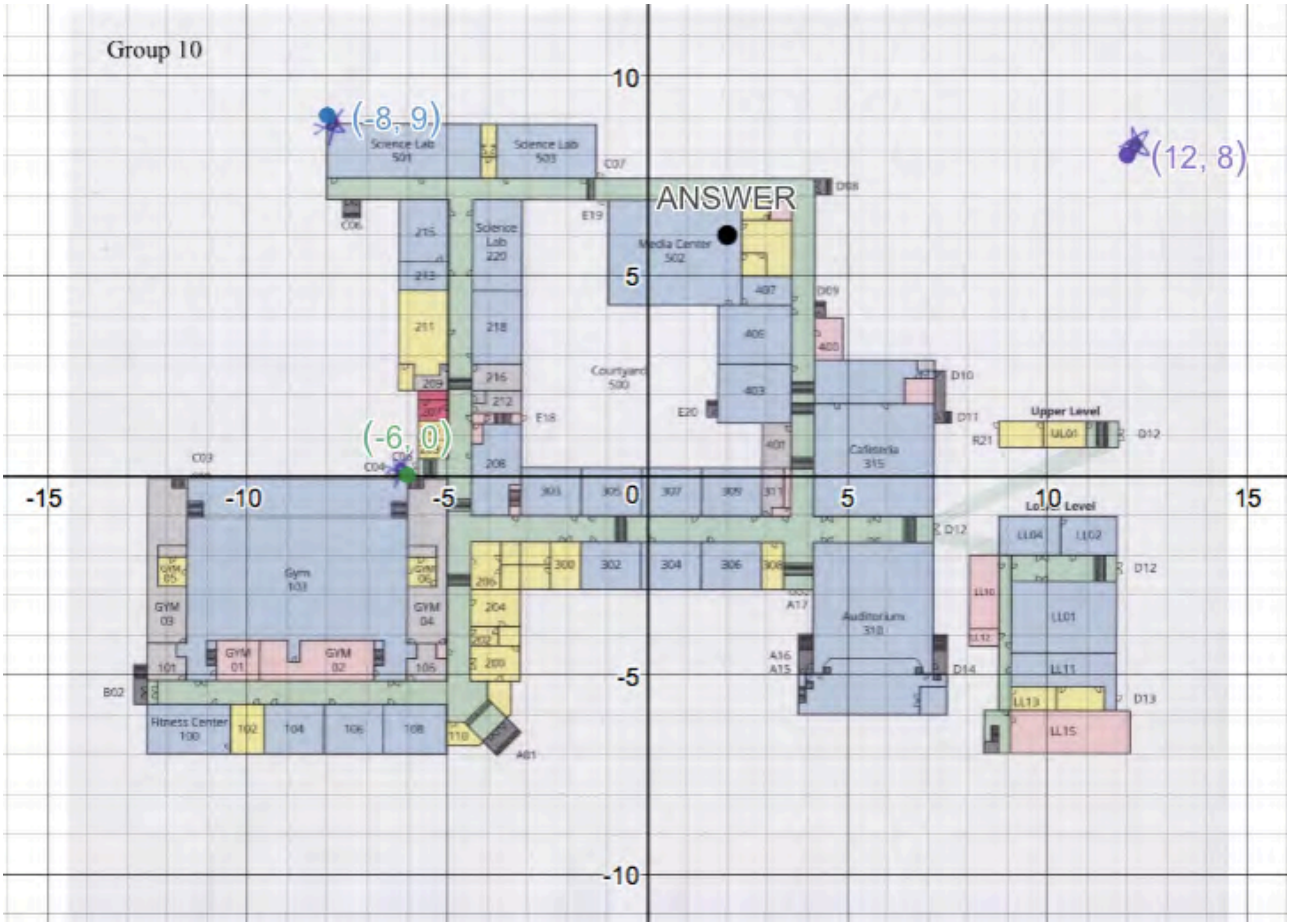
Group 8



(5, 8)
(-5,0)
ANSWER
(6, -10)
Science Lab 501
Science Lab 503
Media Center 502
Courtyard 500
Cafeteria
Auditorium 300
Gym 103
Fitness Center 100
Upper Level
Lower


Group 9
10
5
-5
-10
5
10
(-10, -1)
(-2, 0)
(-3, -8)
ANSWER
Upper Level
Lower Level

Group 10



(-8, 9)
(12, 8)
(-6, 0)
ANSWER
Science Lab 501
Science Lab 503
Science Lab 220
Media Center 502
Courtyard 500
Cafeteria 315
Auditorium 310
Gym 103
Fitness Center 100
Upper Level
Lower Level
10
5
0
-5
-10
-15
15