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Dancing Transformations

Translating, Reflecting, and Dilating Points on the Coordinate Axis

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Lesson Plan
Grade 8

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Lesson Goals

Students typically do not see all the mathematics around them. Transformations are something that exist in art, dance, science, mathematics, and in everyday objects. In this lesson, students will look at dance. It opens students' eyes to mathematics in our world. They may never dance by looking at specific points on an axis, but they will see how our minds use mathematics without even thinking about the numbers. They will see it in performances; they will see it when they see two people walking next to each other on the street holding hands; and they will see it every time they pass a carousel. People will represent points that are simply being transformed.

This activity can be used as an introduction to transformations. It introduces the ideas of transformations, and in future lessons students can expand on those ideas using other transformations, combinations of transformations, and whole shapes.

Teacher Expectations

The teacher expects students to be thinking constantly and making conclusions. She expects them to be devil's advocates and questioning what happens if something changes (ex. The line of reflection).

Student Outcomes

Students choreograph a short dance in which they transform points on an axis. Students represent the points and the transformations, and as they change the type of transformation, they will move across the floor, performing a dance. They will see the power of just a few transformations.

Common Core Standards

CCSS.MATH.CONTENT.8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

CCSS.MATH.CONTENT.8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Common Core Standards for Mathematical Practice

CCSS.MATH.PRACTICE.MP1 Make sense of problems and persevere in solving them.
CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.
CCSS.MATH.PRACTICE.MP3 Construct viable arguments and critique the reasoning of others.
CCSS.MATH.PRACTICE.MP4 Model with mathematics.
CCSS.MATH.PRACTICE.MP6 Attend to precision.
CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.

In discussing and creating transformations, students must use inductive reasoning and look at patterns to make conclusions, and then they must justify those conclusions. When they work collaboratively, they will be questioning each other. They will recognize their errors and see when lack of precision makes their transformation look wrong (ex. students will not be facing each other or they will look too far apart). In some cases, when students have time to extend the activity, they will take what they learned, and apply it to a more complex situation.

Prerequisite Knowledge

Students should have knowledge of how to use a coordinate system. They should be able to graph any point (x,y). They should be familiar with the line y=x.

Time Required

Preparation Time: 15 min **Class Time:** 90 minutes

Materials

Yarn and painter's tape to set up the axes
Something that can be used to play music
Chart paper / Projector to record the table of transformations
Signs with A, B, C, D and A', B', C', D'

Set Up

In advance, you must set up the axes. Tape tends to tear and get broken up on the floor, making it hard to pick up. I would recommend using some colored yarn to represent each axis and the line y=x. Tape the ends down tightly with painter's tape so students don't trip. Most classrooms have tiled floors so you can simply use the tiles to represent units; otherwise you will have to use a ruler and tape to mark off points on the axes. You will have to stack the desks and chairs off to the side, so students have enough room to stand around the axes.

Lesson Activities

Dancing Transformations: Part I - Whole class construction of transformations table

Introduction - Brief Discussion [5 – 10 minutes]

What does it mean to transform something?

You want students to understand that we are talking about change, but also that change can be minor or extreme.

What does it mean to reflect, rotate, dilate, and translate?

You are going to get a wide variety of answers. Discuss one at a time, ask for examples, and then ask about the next one.

What are some examples in the world around us?

- The obvious example of reflection is a mirror – try to get them to think outside the box

*- Dilation could be the windows on computer screens, magnifying images, etc. (*Be careful, one student once gave me the example of breast implants ☺)*

- Translation – every time we move

- Rotation – people on a Ferris wheel

**The point here is to get a lot of different ideas that we see every day, and to get students to come up with them. You can use turn and talk if students don't have examples.*

Discovery - Whole Class Activity [20-30 minutes]

*Explain to students the meaning of *prime* in the context of transformations. A' (pronounced "A prime") is the transformed A, or the *image* of A under the transformation.

- **REFLECTION**

Choose two students to stand on the points A(1, 3), B(4, 0). Give students the signs that correspond to their letter.

Now ask two more students to use the x-axis as a mirror, grab a sign (A' and B') and stand on the points they believe are the reflections in the x-axis. Ask student A and A' to turn and face each other. Are they mirror images of each other? Ask students what that would mean (they should be facing each other and equal distance from the line). Do the same with B and B'. Use inductive reasoning to come up with a general rule: $(x, y) \rightarrow (x, -y)$, and add the rule to the table of transformations (*see page 12).

Note: If you do this on chart paper, you can keep this posted in your classroom throughout the unit so students can keep referring back to it as they work, but make sure students remember to still use the axis to come up with the rules on their own if they forget, so they are not just memorizing.

Repeat this process for reflections in the y-axis with the same points A(1, 3), B(4, 0).

Repeat this process for reflections in the origin with the same points A(1, 3), B(4, 0).
Ask students to compare these new points with reflections in the x-axis or y-axis. You want them to notice that a reflection in the origin is the same as a reflection in the y-axis AND x-axis.

Repeat this process for reflections in the line y=x with the same points A(1, 3), B(4, 0).

Notes:
*Students have a lot of difficulty distinguishing between reflections in the origin and reflections in the line y=x, so the part where they turn and face the line (point) of reflection is very important.
*Make sure to add the general rules to the table (*see page 12) and to record the points used and their reflections. (*see page 14)

If you believe students need more practice, you can add a point C and D, C(0, -2), D(5, 5). *You may want to switch the students who represent A, B, A' and B', so you can get maximum participation, but do not switch too frequently as it will make the dance later in the lesson more complicated.*

- **DILATIONS**
  Use the points that you used for reflection A(1, 3), B(4, 0):
  You are going to dilate this sequence of points by a factor of 2.
  Ask students what they see happening. *You want them to notice that the distance from the origin is what is being dilated.*

- **TRANSLATIONS**
  Now you will translate the points to the left right, up and down by 2 units: A(1, 3), B(4, 0)

- **NOTE: Rotations are a more complex transformation. Students will need more time to determine why the coordinates change the way they do. You can decrease the time you spend on dilations, and translations and cover rotations here, but I recommend you leave rotations for another day.**

Summarizing - Putting it all together [10 – 15 minutes]

Use the chart of points you created on the board, and have students play A, B and A', B'. They can be the same students you used for the first part of the activity or different students.
Put on some music. Have students walk to each set of new points, stand there for a few seconds and then switch to the next transformation. The rest of the class can yell out the next transformation together.

Exit Ticket [2-5 minutes]

*Template included on page 11

- What are two questions you still have about this lesson? If you understood everything about the lesson, then what are two questions you would ask someone else to check if they understood the lesson.
- What are two "What if?" questions?

Note to teacher: these are questions in which students ask what if we changed this; for example, what if we changed the line of reflection to x=2?

Group Activity – Choreography of dance [30 min]

*Activity Sheet included on pages 15-16

- Students are in groups of three or four
- They will each act as a point. They will perform eight transformations on those points, each time transforming the new set of points (not the original points).
- They will record their transformations, the coordinates of the new points, and what they did to the previous points to get the new points.
- If they finish early, they can plot their dance on a coordinate axis, use a third type of transformation, combine transformations, use a different line of reflection, or determine the rules for a 180 degree rotation.

Presentation [10 min]

You can choose two groups to present their dance. Have the presenter for that group summarize their process and choose two transformations to talk about. They should discuss the original coordinate, the new coordinate and how they knew that transformed the point accurately (ex. dilation by 2 should show that the distance from the origin doubled)

Closure - Summary [5 min]

Ask student what operations we use for each transformation:
Reflection – negative signs (except for the line y=x)
Translation – addition/subtraction
Dilation – multiplication/division

Essential Questions

- How do our feet determine the transformation of points on the coordinate axes?
- Can we use patterns to help us determine the transformations of these points?

Extensions

Students can determine what would happen if we changed the line of reflection or if we dilate by a negative number.

Differentiation Options

In the first 45 minutes, you can use a different co-teaching method if you have a second teacher in the room. For example, if you have enough room, you can use station teaching with smaller axes, and you can have one teacher working with dilation, one with reflection, and students can work on translation independently. You can then come together as a group to combine all the points into one dance. You can also do alternative teaching, and at the end, the classes could come together to show their dances to the other part of the class. In the alternative room, you may look at one point at a time.

Use colors to help organize information. For example, use a different color to denote different transformations.

In the group activity, you may consider having some groups plot their work before they try doing it physically on the axes themselves.

The group activity allows students to go beyond the material covered in the lesson if they finish early; you can allow students to go even further by looking at rules for rotations of 90 and 270 degrees.

Lesson Notes and Suggestions

Students have a tendency to complain about standing. You can allow students who are observing to sit on the floor, or if you have chairs without desks, they can sit in the chairs. It is inconvenient to have desks so stack those off to the side.

Consider holding off on showing students notation. The timing is tight and sometimes adding notation to the mix before the concept can make students feel less confident because it is so much to remember simultaneously.

I used the same points for each transformation because they can see the differences more clearly. They will be less likely to mix up certain transformations.

You can either choose group roles (*see page 17) for the students or you can have them choose roles when they get into their groups.

Common Student Missteps

- Students often mix up reflection in the line y=x and reflection in the origin. Make sure they see that they can draw points to make sure they are doing it correctly.
- Students try to memorize the rules, and there are so many. Make sure they know how to use the coordinate axes to determine if the rule they are using is accurate.
- Students may confuse the group activity with what we did in class. In class we always transformed the original points whereas in the group activity we took the previous points and transformed those. You may want to do a brief demonstration with a single point. I changed the group activity to help their dances flow a little better, but feel free not to make that change.

For Future Study

Understanding these basic rules for transforming points will allow students to see relationships between transformations. For example, they will understand that a rotation of 180 degrees is equivalent to a reflection in the origin. They will also see that a reflection in the origin is equivalent in the x-axis and y-axis. Students will proceed to transform whole shapes as opposed to just single points. In later years, they will use these same rules to transform linear, quadratic, and exponential functions.

Exit Ticket

NAME: _____

What are two questions you still have about this lesson? If you understood everything about the lesson, then what are two questions you would ask someone else to check if they understood the lesson.

What are two “What if?” questions?

Exit Ticket

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What are two “What if?” questions?

Transformations Table

Transformation	Rule	Example
Reflection in x-axis	(x, y) →	(1, 3) →
Reflection in y-axis	(x, y) →	(1, 3) →
Reflection in origin	(x, y) →	(1, 3) →
Reflection in y=x	(x, y) →	(1, 3) →
Dilation by k, where k=2	(x, y) →	(1, 3) →
Translation left by h=2	(x, y) →	(1, 3) →
Translation right by h=2	(x, y) →	(1, 3) →
Translation up by k=2	(x, y) →	(1, 3) →
Translation down by k=2	(x, y) →	(1, 3) →

NAME: _____

Transformations Table Handout

Transformation	Rule	Example
Reflection in x-axis	(x, y) → (x, -y)	(1, 3) → (1, -3)
Reflection in y-axis	(x, y) → (-x, y)	(1, 3) → (-1, 3)
Reflection in origin	(x, y) → (-x, -y)	(1, 3) → (-1, -3)
Reflection in y=x	(x, y) → (y, x)	(1, 3) → (3, 1)
Dilation by k	(x, y) → (kx, ky)	(1, 3) → (2, 6) k = 2
Translation left by h	(x, y) → (x - h, y)	(1, 3) → (-1, 3) h = 2
Translation right by h	(x, y) → (x + h, y)	(1, 3) → (3, 3) h = 2
Translation up by k	(x, y) → (x, y + k)	(1, 3) → (1, 5) k = 2
Translation down by k	(x, y) → (x, y – k)	(1, 3) → (1, 1) k = 2

Table of Points

		A (1,3)	B (4, 0)
		A'	**B'**
$r_{x\text{-axis}}$	(x, y) →		
$r_{y\text{-axis}}$	(x, y) →		
r_{origin}	(x, y) →		
$r_{y=x}$	(x, y) →		
D_2	(x, y) →		
Translate left 2	(x, y) →		
Translate right 2	(x, y) →		
Translate up 2	(x, y) →		
Translate down 2	(x, y) →		

**Note, each time you are transforming the original points A, B, C, and D*

Group Work Activity

Names: _____

Your goal is to choreograph a dance using your knowledge of transformations. You will each represent one point. You will agree on a series of transformations you will perform on all points, then you will dance your transformation. Be prepared to present your dance.
**You can quietly play music*

REQUIREMENTS:
1. You must fill in the table at each step, indicating both your transformation and the new set of points and what you did. *Note: that you are performing your transformation on the points in the previous step not on the original points!* ***This is a little different from what we did as a class.***
2. You must perform a minimum of eight transformations and 6 *different* transformations (ex. $r_{y=x}$ is different from $r_{y\text{-axis}}$)
3. You must use at least two of the three transformations (reflection, dilation, translation)

STEP 1:
Choose your original points, one for each of you.

STEP 2:
Create a small axis on the floor

STEP 3:
Fill in your table

Sample Table of Points

Transformation	A(2, 3)	B(-3, 1)	C(0, 5)	D(-1, -4)	What did you do to your point?
$r_{y=x}$	*(3, 2)*	*(1, -3)*	*(5, 0)*	*(-4, -1)*	*Switched x and y*
Translate left 2	*(1, 2)*	*(-1, -3)*	*(3, 0)*	*(-6, -1)*	*Subtracted 2 from x*

Your Table of Points

Transformation	A	B	C	D	What did you do to your point?

STEP 5:
Practice your dance

STEP 6:
Prepare your presenter:
You will talk about your process and then choose two transformations from your table and explain what you did to your points with each transformation and how you know you did it correctly.

STEP 7:
If you finish early you can do one or more of the following:
a) Draw each stage of your dance by plotting the four points on the coordinate axes on paper and then the new points after each transformation. Make sure to label your points at each step.
b) Add another set of points that combines two or more transformations.
c) Add another transformation that is a reflection over a line that is not the y-axis, x-axis, y=x.
AND/OR
d) Determine a rule for rotating a point, (x, y), 180 degrees around the origin

Group Work Roles

Facilitator	*Recorder*	*Questioner*
You will take charge of the group. Make sure every member stays on task and is participating. Keep your group organized and ensure everyone understands all the material.	EVERYONE is required to take notes, but you will be responsible for recording specific questions that come up which your group is not sure how to answer.	You are responsible for asking questions that force your group to consider knew possibilities. (ex. What if …?)
Presenter	*Devil's Advocate*	
You will be responsible for presenting your group's work to the class. You must speak loudly, make eye contact, and ask for questions. If you need assistance you can ask your other group members, but you are the "teacher".	You will be responsible for asking "Why" questions. You want to make sure every group member can describe the reason your group's answers work.	

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