## Abstracts

## Plenary Talks

Persi Diaconis, Mary V. Sunseri Professor of Statistics and Mathematics, Stanford University The Magic of Charles Peirce
C.S. Peirce was an amazing fellow: founder of American philosophy (with O.W. Holmes, W. James, and J. Dewey), mathematician, statistician, geologist, and physicist. He also invented (bad) card tricks that could go on for close to an hour(!). Hidden in these are some terrific new principles. I will try to bring all of this to life, bringing this brilliant, unorthodox fellow and his magic into the 21st century. This is joint work with Ron Graham.

## Manjul Bhargava, R. Brandon Fradd Professor of Mathematics, Princeton University The Magic of Number Theory

Number theory (i.e., the study of whole numbers) is full of beautiful and surprising patterns and structures - some of which are downright magical!
In this talk, we discuss a few of these structures that have inspired some remarkable magic tricks over the years - and at the same time have also brought to light many simple unsolved problems that have now become central subjects of research by modern number theorists.

## Closing Presentation

## Art Benjamin, Harvey Mudd College <br> \section*{The BINGO Paradox}

Paradoxically, when a large number of people play BINGO, it is much more probable that the winning card has a horizontal BINGO instead of a vertical BINGO. We prove this result in the asymptotic case that when every possible BINGO card is being used, then horizontal wins beat vertical wins by a better than 3 -to- 1 margin.

## Research Talks

Max Alekseyev, George Washington University
On Partitions into Squares of Distinct Integers whose Reciprocals Sum to 1
In 1963, R. L. Graham proved that all integers greater than 77 (but not 77 itself) can be partitioned into distinct positive integers whose reciprocals sum to 1 . He further conjectured that for any sufficiently large integer, it can be partitioned into squares of distinct positive integers whose reciprocals sum to 1 . We establish the exact bound for existence of such representations. Namely, we prove that 8542 is the largest integer that does not have such a representation.

Michael Allocca, Muhlenberg College

Bugs, Braids, and Batching

Evolving bacteria, assigning tasks, and braiding hair: find out what all these have in common and learn about super fun mathematical games whose play correspond exactly to these topics. We will discuss game playing strategies for both linear and circular sorting games and show some very deep mathematical ideas that are illuminated by these games.

## Roger Antonsen, University of Oslo <br> Mathematical Explorations and Visualizations with Processing

This talk is about how programming can open up the world of mathematics to both insightful exploration and beautiful expression. With only a few lines of code, mathematical structures become accessible for engaging visualizations and deeper understanding. Keywords: Curves, Envelopes, Primes, Permutations, Partitions, Cellular Automata, Celtic Knots, and More!

## Robert Bosch, Oberlin College <br> Numerically Balanced Dice

We discuss what it might mean for a die to be numerically balanced, explore connections with magic squares, and present integer programming models that can be used to design numerically balanced dice (including d20s, d30s, d48s, and d120s).

## Spencer Congero, University of California, San Diego

Losing at Checkers is Hard
We prove computational intractability of variants of Checkers:
(1) deciding whether there is a move that forces the other player to win in one move is NP-complete;
(2) Checkers where players must always be able to jump on their turn is PSPACE-complete; and
(3) cooperative versions of (1) and (2) are NP-complete.

We also give cooperative Checkers puzzles whose solutions are the letters of the alphabet.

## Yossi Elran, Davidson Institute of Science Education, Weizmann Institute of Science <br> Should we call them Flexa-bands?

A Flexagon is a flat, folded paper polygon with many faces. A half-twisted band is produced by half twisting a strip of paper a few times and joining together its ends. We will show how flexagons and half-twisted bands, frequently used in math magic, share many intriguing properties.

## Darren Glass, Gettysburg College

Chutes and Ladders without Chutes or Ladders
In this talk, we consider various questions related to the board game Chutes and Ladders. In particular, we look at how the expected length of a game on a board with no chutes or ladders changes as you change the spinner size, and also investigate how to optimally place a single chute on the board.

## Henry Guss, Amazon

Platonic Solids of Trivial Symmetry
Presented are two collections of dice designs. The first uses a few colours to cover the shapes such that any orientation of the die reveals a unique pattern of colors when observed from above, while the second achieves this goal using a single pattern that has only trivial symmetry.

## Brian Hopkins, Saint Peter's University <br> Magic Configurations

A configuration is a system of lines and points satisfying certain conditions; Fano, Pappus, and Desargues have configurations named for them. When can we apply the magic of magic squares to these objects? Various methods allow us to find some answers and pose more questions.

## Tanya Khovanova, MIT

## Coins and Logic

I will talk about some old and new coin-weighing puzzles, and their connection to logic and information theory. The new puzzles are formed around an idea of coins that are capable of changing their weights. We will solve some of the puzzles and try to build a parallel to knights and knaves as well as sane and insane people.

## Luis Lafuente, University of Cádiz

## Counting Necklaces by Gilbreath Shuffling

A Gilbreath permutation is a permutation that corresponds to a Gilbreath shuffle. Diaconis and Graham have shown the interest of cyclic Gilbreath permutations. In this talk, based on work by Diaconis, McGrath and Pitman, I will show a natural bijection between Lyndon words with restrictions and cyclic Gilbreath permutations.

## Joseph Malkevitch, York College (CUNY) <br> Rectilinear and Polyomino Knot Projections

The projection of a knot can be chosen as a 4 -valent plane graph, which can be drawn as a polyomino or rectilinear polygon. Interesting questions arise from looking at the number of faces with $i$ sides and valences of vertices associated with such "cut-through" Eulerian circuit embeddings of knots.

## Todd Mateer, Howard Community College (Talk presented by Ricardo Teixeira) <br> A Magic Trick Based on a Double Error Correcting Code

This paper extends a Hamming code magic trick published earlier by the author to illustrate a $(10,3,5)$ double error correcting code.

## Liz McMahon, Lafeyette College

## SET and Simulation: All Sets are the Same, but not Really

What is the expected number of sets in the second layout of SET cards, after a first set has been removed and replaced? Simulations suggest that the answer depends on how you choose the first set, despite the fact that all sets are affinely equivalent. We will explore this mystery.

## John McSweeney, Rose-Hulman Institute of Technology Optimal Strategies for Straights in Yahtzee

In the game Yahtzee, a player rolls five dice and can re-roll a strategically chosen set thereof. In this work we determine optimal strategies for obtaining a straight - a sequence of four or five consecutive dice - by modeling the game via a time-dependent Markov Chain.

## Alex Meadows, St. Mary's College of Maryland

Vennim!
Vennim (first introduced by Paul Salomon) is a variant of Nim in which players may take from more than one pile, based on a Venn diagram. It is also a new way to play Nim on a simplicial complex. We discuss some preliminary results and interesting questions.

## David Molnar, Rutgers University <br> Wiggly Games and Burnside's Lemma

A genre of board games features collections of tiles with arcs on them, which are placed together to form paths. We refer to these as wiggly games. Tsuro is a well-known example. I will show how to calculate the total number of tiles possible in such a game, modulo symmetry.

## Colm Mulcahy, Spelman College <br> Sum Effect with Free Selection of Any Number of Cards

The deck is shuffled, and several spectators remove cards while the mathemagician's back is turned. The sum of the values of the selected cards is reported. Despite not knowing how many cards were selected, the mathemagician names all of the cards.
This generalizes an invention of Colm Mulcahy (published online in 2008) where 2 cards are selected and the sum of their values is reported. See http://www.maa.org/community/maa-columns/past-columns-card-colm/additional-certainties. Unlike that creation, which used Fibonacci numbers, or generalizations thereof, this uses a different mathematical principle, that of dissociated sets.
In reality, despite fair seeming shuffling, the top stock remains unchanged. The selected cards may be any subset of the original top 5 cards. The $2^{5}=32$ possible sums are all distinct, if appropriate values are used, and the suits of the five cards in question are memorized to facilitate complete identification.

## David Nacin, William Patterson University <br> Finite Group KenKen

KenKen is a popular type of paper puzzle, usually done over subsets of integers. We discuss similar constructions over the elements of a finite group. We focus on how the changing between different groups can affect the strategies involved, the solvability of the puzzle, and the number of solutions.

## Simon Norton, Cambridge University <br> Bridge Deals and Magic Squares

The suit distributions between players in bridge deals define "submagic" squares, where the rows and columns add up to 13. Bridge expert Ely Culbertson's "Law of Symmetry" suggested that rows tended to be permutations of columns. We discuss the occurrence of this and other "higher magic" properties in such squares.

## Miguel Palomo, Universidad Politécnica de Madrid

## Sudoku Ripeto and Custom Sudoku

In this talk I will introduce Sudoku Ripeto and Custom Sudoku, two new Sudoku variants that I have created. They seem to be the first ones to feature repeated symbols. I will discuss the mathematical and algorithmical foundations guaranteeing uniqueness of solutions, polynomial-time resolution and relevant classification by difficulty.

## James Propp, Mathematical Enchantments

## Engel Machines

In 1974 Engel found a way to teach probability to kids using a device that presaged work of Diaconis (as well as the theory of chip-firing and sandpile groups) and paved the way for Tanton's Exploding Dots. Some Engel machines will be on display during MOVES; I'll explain why they work. You'll also get to experience Engel machine computations musically.

## Jason Rosenhouse, James Madison University <br> The Saga of the Hardest Logic Puzzle Ever

The "Hardest Logic Puzzle Ever" was first presented by George Boolos in 1996. Since then it has spawned a small industry of variants and philosophical discussion. We shall consider the original puzzle and solution, attempts to make it harder still, and recent versions based on nonclassical logics.

## Karl Schaffer, De Anza College <br> Edgy Puzzles

Countless puzzles involve decomposing areas or volumes of two or three dimensional figures. We will examine puzzles in which the edges of various symmetric figures like polyhedra are decomposed into multiple copies of smaller graphs, and see their relationship to representations by props or body parts in dance performance.

## Ann Schwartz

## Flexagon Discoveries: Going Beyond the Tetra- and Hexaflexagon

After a quick presentation of early flexagons, I will show new ones made from straight strips folded into shapes that include triangles, six-pointed stars, squares, irregular hexagons, and rings. Along with these discoveries come some surprising characteristics appearing over this diverse range of complex flexagons.

## Chee Wei Tan, City University of Hong Kong

Algebra Game: Mathematics, Algorithms and Use in Mathematics Festival
We shall talk about the Algebra Game Project (www.algebragamification.com) based on an inspiration of gamifying algebra by Terence Tao (mathematician at UCLA) on the puzzle mathematics and its software. We shall also talk about how the Algebra Game mathematics has been used in the Julia Robinson Mathematics Festival in Hong Kong.

## Ricardo Teixeira, University of Houston - Victoria

Data Transmission Algorithms Applied to Magic Tricks
World-famous magicians perform acts heavily based on mathematical concepts, but rarely do the explanations get proper attention. In this talk, we will cover tricks whose explanations involve error detection and correction methods, as well as number systems.

## Robert Vallin, Lamar University

Penney's Roulette and Martingales
Penney's Game involves choosing a three-outcome sequence of coin flips. The game is non-transitive (no matter what Player I's choice of outcome is, Player II can put the odds in her favor). We will play this game on a roulette wheel and use martingales to determine the odds in Player II's favor.

## Jonathon Wilson, Ferris State University

Entropy of Non-rectangular LEGO Bricks
Let $T(n)$ be the number of ways to connect $n$ LEGO bricks of the same type together. In this talk, we discuss the growth rate of $T(n)$ and describe upper and lower bounds on the entropy of non-rectangular LEGO bricks shaped like the letter L.

## Peter Winkler, Dartmouth College <br> Probability Magic

The tools of probability theory are so powerful that many problems can be solved "magically" without pencil or paper-or computer. And perhaps, when you figure out how to solve a problem "in your head," you can better understand why the answer is what it is. We'll examine some intriguing probability puzzles and see how to get to their solutions by pure reasoning.

## Wing Hong Tony Wong, Kutztown University of Pennsylvania

## Graph Coloring Games, Col, and Nimbers

Inspired by MOVES 2015, we study a graph coloring game that resembles the game of Col. We examine various families of graphs and determine which player has a winning strategy. Examples of such families include paths, cycles, rectangular grids, triangular grids, and Cayley graphs. Nimbers are involved in certain proofs.

## Family Activities

## Walker Anderson, Central Bucks West High School

## Fun with Fillomino

Fillomino is a grid logic puzzle created by Nikoli Co., Ltd. In a Fillomino puzzle, a grid of connected squares (often rectangular in shape) must be split into polyominoes, or shapes made of connected squares. A given number in the grid must be equal to the area of the polyomino it is inside of. Polyominoes with the same area must not be adjacent. All Fillomino puzzles have a unique solution that can be found logically, without guesswork.
This presentation will begin with a description of the rules of Fillomino. Afterwards, three solving techniques for Fillomino will be presented. Finally, attendees will have the opportunity to solve Fillomino puzzles on their own. The presenter will be available to provide clues and assistance.

## Jennifer Austin, The University of Texas at Austin <br> Not Just a Knot

Imagine you hook a big fish. It leaps out of the water. Your line knots. SNAP! Next imagine you are sitting in math class, dust is flying, and you are burning lead taking notes. Oh no! There is a knot in your DNA. How did this happen? What happens next?

## Hossein Behforooz, Utica College

## A Practical Workshop on Magic Squares

In this workshop, we will introduce you to a very fun part of recreational mathematics. It is called Magic Squares. Yes it is magic and it is FYE which means for your Mathematical Entertainment and Adventure. After this display and workshop, you will go home with many tables with numbers. Most of these magic squares have many interesting and amazing properties and that is why it is called MAGIC SQUARE. This workshop is open to any grade from elementary school to college students and also parents. Come and join us and have fun. You will love it! Math is FUN.

## Silvia Benvenuti, University of Camerino

## Geometries in motion!

Who said that studying geometry is boring?! Probably he has always done this using ruler, square and compass. But what if instead we use as the main tool our body? If you are ready to jump, tie, put yourself upside down... then we can discover it together!

## Nancy Blachman, Julia Robinson Mathematics Festival <br> Recreational Mathematics (Really!)

I invite you to puzzle with squares and cubes. First, we'll try tiling a rectangle with squares of different sizes. Can it be done? Yes - and all it requires is a bit of ingenuity, addition and subtraction. Next we'll all become city planners and figure out where to build skyscrapers of different heights. This is a great little logic puzzle. You'll receive a Puzzling with Squares booklet and cubes with which to build a city.

## Skona Brittain, SB Family School

## The Most MatheMagical Number

A few magic tricks involving the most MatheMagical number will first be presented, then figured out or revealed, and then mathematically analyzed. Participants will make their own MatheMagics cards for an unusual one of the magic tricks and practice performing with them.

## Ken Collins, Charlotte Latin School <br> Introduction to the game of SET

The game of SET has been popular for 25 years. Young children through adults can play the game. It can be enjoyed for its own sake and also for the mathematics it uses. Come see for yourself!

## Sasha Fradkin and Allison Bishop, Main Line Classical Academy

## Functions come to life in Funville Adventures

We propose a set of activities based in a world of personified functions. Participants will be introduced to various function characters from our upcoming book Funville Adventures and solve who-done-it style puzzles. They will explore concepts like commutativity, invertibility, and periodicity. There will be puzzles of various levels of difficulty.

Milenas Gonzalez and Jennifer Woodruff, Bank Street College of Education
Math with Young Children: Using an Understanding of Math Development to Play Mathematical Games

Using research on the development of children's mathematical thinking, we will present a variety of games that can be played with children aged 2-8 at home (or standing in line at the supermarket!) to support their learning in subitizing, measurement, counting, comparing, patterning, addition/subtraction, and geometry. You won't want to miss Mr. Mix-Up!

Jane Kats and Tatiana Ter-Saakov, Moscow Center of Continuous Mathematical Education, Independent University of Moscow
Chickpea-toothpick building set
Among various building sets chickpea-toothpick set is our favourite. It is a big hit in every Math Festival in Moscow. Chickpeas give variability - you can chose any angle to insert the toothpicks - and sturdiness. They also have much better chance to end up as part of the construction than marshmallows.

## Rebecca Klemm, NumbersAlive!

Polygon Puzzles-Assembly and Creation For All Ages
Young puzzlers, have fun and be challenged with Number Linx puzzles and memory game cards linking numbers, shapes and patterns. Older puzzlers, come assemble or create Puzzling Polygons where each piece has the same number of sides as the resulting polygon. Then consider a proof of the Puzzling Polygon Conjecture.

Ed Lamagna, University of Rhode Island
Playing with Half a Deck... Plus One!
A card trick attributed to Gergonne provides a vehicle for understanding positional number systems. Participants will learn how to perform this dazzling trick and the mathematics behind it. While based on ternary numbers, the trick is readily adapted to other bases and operates similarly to the radix sort algorithm.

## Colm Mulcahy, Spelman College

## Additional Certainties

Imagine having two people pick cards from a deck and your being able to identify both cards when only told what their values add up to. Now imagine doing the same trick with several people even if you don't know how many people participate! We'll demonstrate both tricks and then explain how mathematics can be used to make both possible.

## Maura Murray and Julie Belock, Salem State University <br> Sierpinski Gaskets and the Chaos Game

Participants will explore the construction of several self-similar fractals using simple recursive rules. One famous example will be the two-dimensional Sierpinski Gasket. We will then learn about the Chaos Game and play it using a single six-sided die and transparent paper with magical results!

## James Propp, Mathematical Enchantments

Fire When Ready
Simple rules can lead to surprising outcomes when lots of people are following those rules. We'll explore what happens when people pass around tokens following a simple rule: you collect tokens until you have enough to evenly distribute to your neighbors. This may cause a chain reaction!

## Elana Reiser, St. Joseph's College

## The Art of Cryptography

Have you always wanted to learn how to send secret messages to your friends? Learn about cryptography and the art of encoding and decoding messages. We will look at the Caesar Cipher, the Vigenere cipher, and the Homophonic Substitution cipher. Plus, you get a decoder ring!

## David Richeson, Dickinson College

Sugihara's Impossible Cylinder
In summer 2016 Kokichi Sugihara posted a video online of some "impossible" objects that look like one shape head on and a different shape in the mirror. It quickly went viral. In this family activity we present paper templates for making your own impossible cylinders.

## Ann Schwartz

## Playful Geometry: A Flexagon Workshop

A repeat of the MoMath workshop last August, this family activity will teach children and adults how to make two hexaflexagons and two square 8 -flexagons. If time permits, an exciting flexagon discovery will be taught to the group. Flexagons are fun toys and the MoMath workshop was a success.

## Melissa Silk, University of Technology Sydney <br> Binary Bugs Family Workshop

Explore the complexity generated by the interaction of two simple systems: a randomly created 2D binary pattern and structural 3D paper folding. Enhance the geometry of the 3D pattern by coin tossing to determine the black and white design. Discover how biomimicry meets mathematics in a joyful math-making challenge.

## Ron Taylor, Berry College

Two Family Oriented Color Addition Games
In this presentation, we will introduce two family style sequential games whose rules are mathematical in nature, though no explicit mathematics is necessary during game play. Both games are based on color mixing rules which can yield nice geometric visual presentations. One game is played with colored stones and each player can see every other player's hand. Players take turns trying to reduce their pile of stones by making exchanges with a central pile of stones according to the color mixing rules. This game has single player and team variations. The second game is played with special dominoes and each player's set of pieces is hidden from the other players, though the set of available moves is known to all players.
Both games are based on a combination of additive and subtractive color mixing arithmetic, which are distilled into five fundamental rules of the game. These can be explained using a combination of metaphors about finger paints and light bulbs.

## Kira Zelbo, Brearley School <br> Rolling Cube Puzzles

The idea behind the puzzles is that a painted cube rolls and leaves a trail of paint behind. The puzzles ask you to think about the patterns left behind as the cubes roll. What patterns are possible / impossible? What always holds true about the relative placement of the colors? Can you tell by looking at a trail of paint, for example, which colors are opposite each other on the cube and which colors touch at a corner?

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