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Algebra on a Number Line

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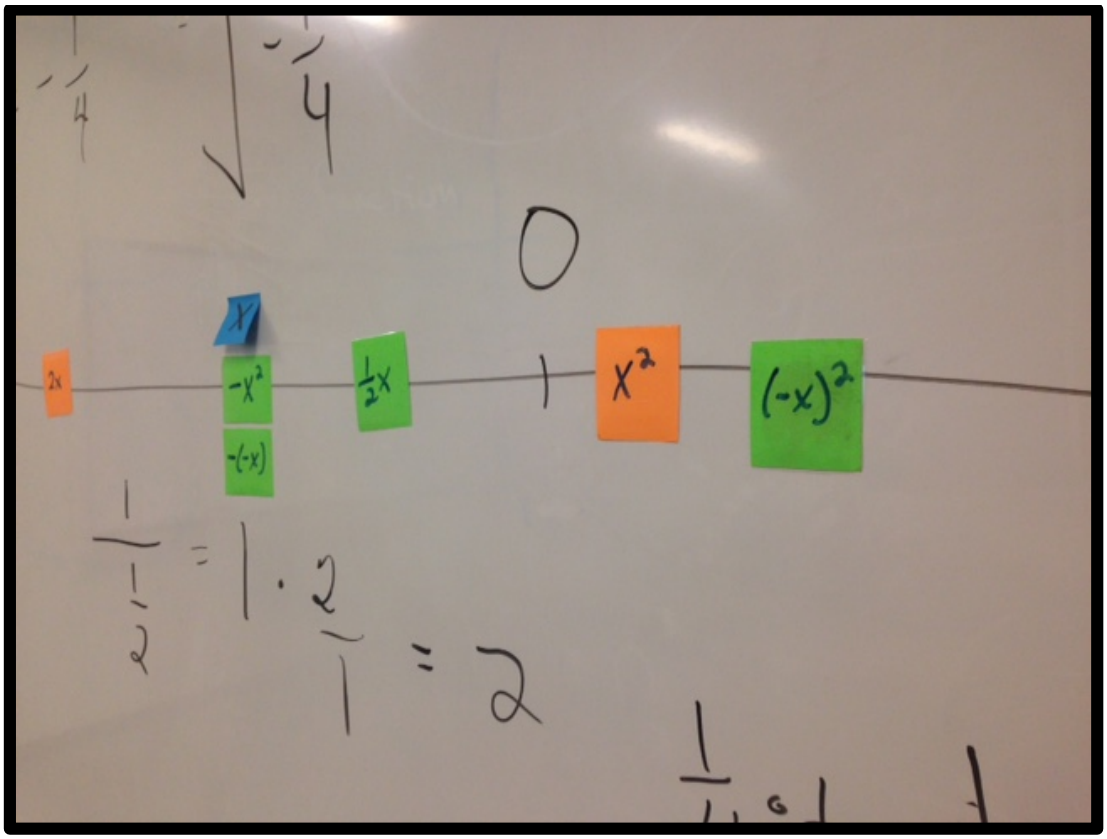

Lesson Plan

Grades 7-8

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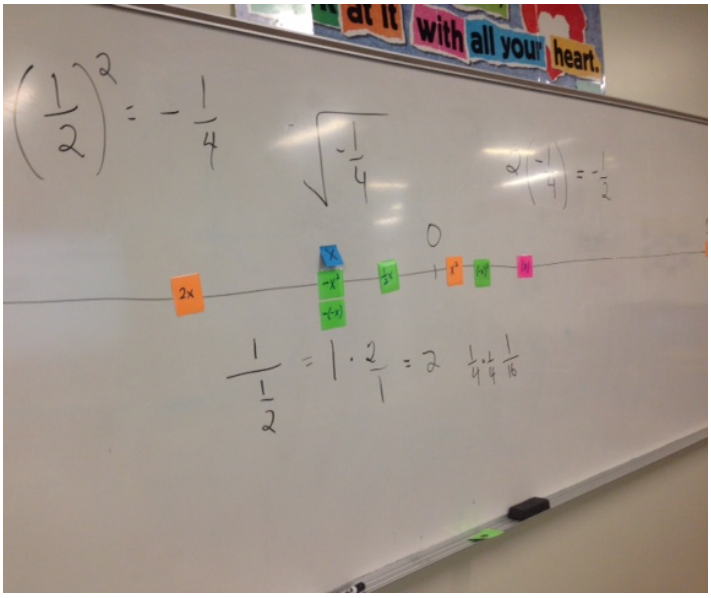
Overview



Students typically use a number line in math class when learning to add and subtract integers or when graphing solutions to one-variable inequalities. The number line, however, can offer much more to a student's understanding of relative position of values and the density of the real numbers.

In this activity, students will take turns placing number values and algebraic expressions at the appropriate locations on a number line (a long string across the room or a line on the board). This is a whole-class activity in which each student takes a turn to participate placing his or her values (magnetic cards or post-it papers) between "benchmark" values such as -1, 0, 1.

Students have the opportunity to get out of their seats and interact with each other about numbers and expressions. Because of the limited supplies needed and easy setup, teachers can provide exposure to values on a number line on a frequent basis.

Prerequisite Knowledge

The students may have some knowledge of variables, basic operations, some advanced operations (such as exponents, square root, and absolute value) locating numbers on a number line, integers, approximating irrational numbers and algebraic expressions. The values and expressions on the cards used in this activity can be adjusted depending on the class's overall level. Additionally, each student can be given a value or expression on his or her individual card that meets his/her level. Because of the wide range of possible cards, this activity can be adjusted for grades 6–11.

Activity Goals

The students will deepen their understanding of numbers and variable expressions' values by physically placing their value on a number line in relation to other values. When using variable expression cards, students will connect the original value of the variable with how the value changes in an algebraic expression (for example, how does the value of x change when it is squared). The students will be responsible for placing their own value card, but they may work in a small group to receive guidance in this task.

The Common Core State Standards for Mathematics

Because this activity lends itself to introducing various algebraic expressions, the above standards are appropriate for grades 7–8; however, the activity can be easily adjusted for grades 6–11.

CCSS.MATH.CONTENT.7.NS.A.1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
CCSS.MATH.CONTENT.7.NS.A.1.B	Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
CCSS.MATH.CONTENT.7.NS.A.1.C	Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
CCSS.MATH.CONTENT.7.NS.A.2.A	Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (-1)(-1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
CCSS.MATH.CONTENT.8.EE.A.1	Expressions and Equations Work with radicals and integer exponents. Know and apply the properties of integer exponents

	to generate equivalent numerical expressions. For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. Know that there are numbers that are not rational, and approximate
CCSS.MATH.CONTENT.8.NS.A.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.
CCSS.MATH.CONTENT.8.NS.A.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue to get better approximations.

Activity Details

Timing [variable]

The Number Line activity is not necessarily a stand-alone lesson. It can be used repeatedly throughout the year to reinforce new concepts or to review. Activity time can be adjusted from just a few minutes to an entire class period.

Materials

- Thin rope or string to reach across the classroom
- Cards with the numbers -1, 0, and 1 (or an alternative method of designating benchmarks)
- Number and expressions cards (for example, a post-it or index card along with a paper clip or clothes pin to be attached to a string, laminated index cards with a sticker magnet attached to the back, etc.)

Activity Description

1. To start the activity, stretch a thin rope or string across the room with previously placed values of -1, 0, and 1 as benchmarks.
2. As an optional warm-up, provide each student with a card with a number on it. The students take turns placing the values on the

number line. These values can be a mix of fractions, decimals, percentages, integers, or irrational values depending on the level of the students. Once everyone has had a turn, remove all cards except the "benchmarks" to clear up space for step 3.

3. To deepen algebraic thinking, the teacher places a card "x" anywhere on the number line. Now the students receive new cards with algebraic expressions, such as $2x$, $1/2\,x$, x^2, x^{-1}, $\sqrt{x}$, $|x|$, $x+1$, $-x$, and many others (see Sample Cards).
4. Then, the teacher can change things up by moving the previously placed "x" card. This encourages a conversation of how should we move our algebraic expression cards accordingly. For example, now that "x" is negative, how does this change the other cards we placed?

Sample Cards

x^2	$-x$	$2x$	$\frac{1}{2}x$	$\sqrt{x}$
$\lvert x\rvert$	$-\lvert x\rvert$	$x-1$	x^{-1}	x^0
$x^{1/2}$	$-\sqrt{x}$	$-(-x)$	$(-x)^2$	$\frac{x}{2}$
$1-x$	$0x$	$\frac{1}{x}$	$-2x$	25% of x

Ongoing Assessment

While students are placing the cards, they get instant feedback and guidance from the teacher and the other students. An ongoing dialogue

is maintained as each student considers where his/her card should be placed in relation to the other values. The teacher can informally assess the performance of individual students and the overall class performance. Blank cards can be available for the teacher to adjust concepts on the fly. The teacher plays a facilitator role in this activity rather than a direct instructor role. The students are responsible for placing their cards and may seek guidance from classmates and the teacher. The teacher may want to have blank cards ready to create new expressions as needed to either extend or remediate a concept.

Possible facilitator questions:

- (After placing the "x" card) Using the benchmark values, how can we determine the value of x? Is it positive, negative, less than 1?
- When placing x^2, will it be larger or smaller than x? How can we tell?
- When placing "$-x$", will the value be less than or greater than 0?
- (Advanced questioning) If $x = 0$, where do we place x^{-1}; if x is negative, where do we place $\sqrt{x}$?

Differentiating the Activity

Cards can be custom-created for each student. Some students may be ready for negative exponent expressions, while some may be challenged with expressions such as "$x - 1$". Asking students to create their own expression cards, which, then, can be exchanged with a partner, can also extend this activity.

Variations

If working with a small class, place blue painter's tape on the floor in a long hallway as the number line. The students stand on the number line holding their cards to represent the value.

For larger classes, the number line can be used as a stop on a station rotation for small groups to take turns.

Resources

http://web.njcu.edu/sites/faculty/dbennett/uploads/mtms2009-04-458a_darley.pdf