

# THE 2016 ROSENTHAL PRIZE *for Innovation in Math Teaching*

# Where Can I Find A Weightless Stick

Dena Lordi



Lesson Plan Grades 7-8



# **Table of Contents**

Lesson Overview Prerequisite Knowledge Lesson Goals	<b>3</b> 3
Assessment of Achievement	4
The Common Core State Standards for Mathematics	4
The Common Core Standards for Mathematical Practice	5
Lesson Details	5
Time Required	5
Materials	5
Optional Materials	6
Class Preparation	6
Activity 1	7
Overview	7
Teaching	7
l ell the Students	8 8
Activity 2	9
Overview	9
Tell the Students	9 Q
Notes	10
Activity 3	11
Overview	
Teaching	11
Notes	13
Lesson Extension Activities	13
Group Task Sheet: Balance Point and Center of Gravity	15
Group Task Data: Balance Point and Center of Gravity	16
Individual Understanding 1: Balance Point and Center of Gravity	/ 17
Individual Understanding 1: Balance Point and Center of Gravity	1/
CC Calculation Tomplete Evaluation for the Teacher	y 18
CG Calculation Template, Explanation for the Teacher	19
Weight and Balance Performance Data	20
Meter Stick Illustration	21



# Lesson Overview

In this lesson, students will collaborate on an experiment to determine the relationship between the mean value of a set of data and the center of gravity of a stick. As this lesson relates a mathematical concept to a physical reality, the students will use conceptual and computational skills as well as physical tools to explore the relationship between the world of mathematics and the physical world.

Specifically, in this lesson, we ask students to recall that the average of a set of numbers is often referred to as the *balance point*. We then ask them to perform an experiment to test if this balance point is the same as the center of gravity. The students must then resolve the conflict between what is mathematically true and what is actually true.

#### Prerequisite Knowledge

To meet the goals of this lesson, students must be able to

- perform basic calculations with rational numbers represented as fractions or decimals;
- calculate the mean value of a set of numbers;
- represent a real-life situation with a numerical or algebraic expression;
- use a meter stick to measure length to the nearest tenth of a centimeter;
- work collaboratively in small groups.

This lesson is designed to enhance a unit on the measures of central tendency for grades 7 and 8. However, a teacher can easily adapt it for lower grades to be an introduction to calculating averages. Alternatively, a teacher can adapt this lesson for high school classrooms to further introduce ideas from calculus, physics, and engineering.

This lesson is not designed to show how the mean value can be used for data analysis; rather, this lesson treats the concept of mean value as a vehicle that can be used to transition between mathematical problem solving and problem solving in the real world.



#### Lesson Goals

After participating in this lesson, students will be able to

- articulate the relationship between the world of mathematics and the physical world;
- demonstrate understanding of the center of gravity and how it relates to the mean value;
- collaborate with their peers to solve a complex problem and make an informed prediction;
- apply the calculation of the mean to a real-world situation and use that calculation to predict the location of the center of gravity.

#### Assessment of Achievement

Students will demonstrate their achievement of the lesson goals by

- participating in whole-class and small-group discussions of key questions;
- collaboratively completing the Group Task document;
- individually completing the *Individual Understanding* document.

#### The Common Core State Standards for Mathematics

<u>CCSS.MATH.CONTENT.7.EE.A.</u> Use properties of operations to generate equivalent expressions.

- 1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

<u>CCSS.MATH.CONTENT.7.EE.B.</u> Solve real-life and mathematical problems using numerical and algebraic expressions and equations

- 3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.
- Use variable to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.



#### The Common Core Standards for Mathematical Practice

- 1. Make sense of problems and persevere in solving them.
- 2. Reason abstractly and quantitatively.
- 3. Construct viable arguments and critique the reasoning of others.
- 4. Model with mathematics.
- 5. Use appropriate tools strategically.
- 6. Attend to precision.
- 7. Look for and make use of structure.
- 8. Look for and express regularity in reasoning.

In collaboratively solving several problems, students will be successful only if they attend to all eight standards for mathematical practice. For example, students should note how attention to precision in measurement can change the value of a result, as can a careful and deliberate use of available tools, including good management of group roles within a collaborative exercise. The lesson assessment demands that students communicate both rational arguments as well as personal understanding of concepts. In addition, the students must recognize that the structure of a familiar calculation can be adapted to apply to a new and different situation.

#### Lesson Details

#### Time Required [90 minutes]

Properly presented with good opportunity for summing up, this lesson will take 90 minutes, a little less than two 50-minute class periods.

#### Materials

- Balsa-wood sticks, 0.5" X 0.25" X 36" with one end marked as a reference point for measurements, one for each group
- Wood blocks to use as fulcrums, one for each group. Mark a center line on the top of each fulcrum to aid in precision.
- ✤ 3.25" clothespins, five for each group
- Meter sticks, one for each group
- Basic calculators, at least one for each group
- Writing utensils



- Group Task documents, one for each group, attached (page 15-16)
- Individual Understanding documents, one for each student, attached (page 17-18)
- Image of Embry-Riddle University Weight and Balance pre-flight document, attached (page 20)
- Image of meter stick to check measurement practices (page 21)

#### **Optional Materials**

To demonstrate center of gravity in non-uniform objects and in moving objects:

- Model airplane
- Shovel
- Projector/computer to show video of Simone Biles vault dismount found online at <u>https://m.youtube.com/watch?v=owHmsXXIsRo</u>
- 1.75" clothespins for additional problem solving

Note: The weight of the balsa wood stick as compared to the weight of one clothespin will be important to the solution of our main problem. We used digital scales in our science department to get an approximation that one balsawood stick has a weight that is 1.3 times that of the clothespins that were used. You may use this number or re-check your specific materials.

#### **Class Preparation**

Have the students arranged and seated in groups of 2-4. Each group should have a large surface (two adjacent student desks or a table) for working. As an optional, pre-lesson exercise, you may have the students view the Simone Biles video and ask them what the announcer is referring to when he says we should "watch her center of gravity." *Why would that be important?* 



### Activity 1 [10 minutes]



#### Overview

In this activity, we quickly review some basic ideas about *averages* of numbers and then introduce the concept of *center of gravity*.

The teacher will lead the class in a calculation of average and a discussion of the average as a balance point. The teacher will then introduce center of gravity as a property that applies to all real-world objects. The class should then discuss what might be some differences between a balance point within a mathematical calculation and the center of gravity of a real-world object.

#### Teaching

Distribute the *Group Task* documents, the printed photographs, the balsa-wood stick, the meter stick, fulcrum, and clothespins to each group.

Have the students complete the calculation: Find the average of the following three numbers: 3, 4, and 11.

Once you've all agreed on the result 6, note that the average of a set of numbers is often called the *balance point* of the set.

Ask: Why do you suppose we would call it that? What are we balancing?

To support student ideas, display the following diagram and explain that the average is the balance point because, if we find the distances between the numbers and the mean, the sums of those distances on each side of the mean are equal. In the diagram, the distances are in red.





You may also relate that in physics and engineering these distances are called *lever arms* and they are related to *torque*, which is a measurement of the tendency of an object to want to rotate. One side of the fulcrum wants to rotate clockwise; one side wants to rotate counterclockwise. Since the object is not moving, we know the torque values in both directions are equal and opposite; the object is balanced. If an object is balanced, the forces acting on it are in equilibrium.

#### Tell the Students

"As mathematics students, we live in two worlds: the math world and the real world. While the idea of average as balance point seems to make sense in the math world, today we will investigate whether that idea works in the real world. Center of gravity is the real-world version of a balance point. The center of gravity (CG) of an object is the location of the average weight of the object. It is the one place where we can support all the weight of the object. At the CG, the object is balanced."

Demonstrate the principle of the center of gravity by showing it in a few objects: a shovel and a model airplane are good examples.

Center of gravity is particularly important for pilots. In fact, before each flight, a pilot needs to perform calculations involving the center of gravity to make sure the weight distribution on the airplane will keep the plane stable. Show students the attached example from Embry-Riddle Aeronautical University (page 20).



# Activity 2 [40 minutes]



#### Overview

In this activity, students will first agree on a level of precision for measurement, then—using balsa-wood sticks, clothespins, a meter stick, a fulcrum, and sets of three numbers—make observations about the center of gravity and the average of the numbers. Ultimately, they will use their observations to try and predict the value of the center of gravity.

#### Teaching

#### Tell the Students

"We are now going to test to see if the balance point or average really is the same as the CG in the real world. The first thing we will notice is that, in the real world, we will have to measure distances. In the math world, we can count or calculate to get exact values. In the real world, we need to measure, and measurement is always an approximation."

Show the students the picture of the meter stick and have them state the position of the arrow. Note the level of precision is important. This measurement is correctly, depending on your level of precision, 50 centimeters when measured to the nearest 10 centimeters, 46 centimeters when measured to the nearest centimeter, or 46.3 centimeters when measured to the nearest tenth of a centimeter (nearest millimeter). Ensure that the students understand that we will be measuring to the nearest tenth of a centimeter, so, for our experiment, 46.3 cm is correct.



Instruct the students to create a workspace at the table, assign roles within the group, and complete the experiment described in the Group Task document. The first task and the beginning of the second group task are best delivered as a guided activity with the teacher demonstrating and the students completing the task and measuring with their own equipment. It is important that you take the students through the first prediction and encourage them to use evidence to make the prediction. After that, let the students, in their groups, complete the remaining group tasks on their own.

Finish this activity by giving the groups time to carefully record their group answers to the discussion questions on both the front and back sides of the Group Task document.

#### Notes

It is important to demonstrate practices that will result in the best precision in measurement. Ask the class for suggestions about how to make the measurements more precise. Student ideas may include the following:

- use the center-line mark on the fulcrum as the location of the CG;
- always measure with the marked end of the balsa-wood stick lined up with the zero on the meter stick;
- mark the CG on the meter stick with a pencil before you take it off the fulcrum to measure.

The students will know what it means to create a hypothesis from their science class, so recall this when talking to them about predicting the location of the CG. You will need to teach and model articulating and justifying a prediction.



# Activity 3 [30 minutes]

#### Overview

In this activity, the teacher will lead a discussion that will resolve the questions:

- Why was the CG different from the average of the numbers in the first two data sets?
- What is causing this difference?
- Why was the CG very close to the average of the numbers in the third data set?
- How can math help us resolve this conflict between what we expect and what we observe?

#### Teaching

Distribute the Individual Understanding documents to each student.

By asking the questions above, have the students recall their group's results from the previous activity and offer their theories. You can expect that students will cite—among other ideas—precision, density of the clothespins and the balsa wood, and ultimately the weight of the balsa-wood stick as the cause of their inability to accurately predict the location of the CG by just using the average of the numbers on their lists.

Ask the students to further consider how we might account for the weight of the stick in our calculation. Explain that, in the world of mathematics, we can consider that the balsa-wood stick represents another clothespin because one clothespin and one stick weigh about the same. This is like having a new, virtual, invisible clothespin somewhere on the stick. But where?

(Following this lesson, you might ask your students, "In what other situations does math allow us to see something that is otherwise invisible?").

The following diagram uses the data from the first data set on the student Group Task document. Use this diagram, or one like it, to assist the explanation. The black arrows indicate the clothespins pushing down in the stick. The red arrow indicates the weight of the stick pushing down at its original center of gravity—when the stick was empty. This is like us supporting other objects, like the shovel in the earlier activity, at their center of gravity.



That is, we can consider that all the weight of the stick is located at the stick's CG. This must be the location of our new, virtual clothespin! The numbers in the diagram represent the distances from the left end of the stick. Note that the distance marking the stick's CG is only an example. This is a measurement, and yours may be different.



Using this diagram, we can now see that to find the new center of gravity (marked in green in the diagram) we need to calculate a new average with four numbers instead of three. Here is an example of the calculation for the values in the first data set. This is a simple calculation of the average of four numbers.

$$\frac{10+20+60+45.3}{4} \approx 33.8$$

Take the students through this calculation using the templates on the Individual Understanding 2 document (page 18) and the accompanying annotated diagram for the teacher (page 19). Note that these templates also account for the frequency of each data value. In other words, there is one clothespin at 10 cm, so we multiply 10 by 1, and so on. The calculation now looks like

$$\frac{1 \bullet 10 + 1 \bullet 20 + 1 \bullet 60 + 1 \bullet 45.3}{1 + 1 + 1 + 1} \approx 33.8$$

The calculation of this new average will be much closer to the CG that the students measured when they first worked with this data set. In addition, including frequency in the explanation will allow students to more easily understand the next step of the process, in which we make our calculation even more accurate. Show the students how to increase their accuracy by noting that the weight of the stick is actually about 1.3 times the weight of a clothespin. *How will this change our calculation? Our stick is about 1.3 clothespins.* 



# $\frac{1 \bullet 10 + 1 \bullet 20 + 1 \bullet 60 + 1.3 \bullet 45.3}{1 + 1 + 1 + 1.3} \approx 34.6$

You can see this calculation differs from the previous one only in that we have changed the frequency of the value for the stick from 1 to 1.3: The weight of the stick has the same effect on the measured center of gravity as 1.3 clothespins would.

For closure, have the students complete the vocabulary matching and the three questions on the Individual Understanding document. The activity ends with a large class discussion of the original question to the groups:

"Is *the balance point* an accurate term to use for the average of a set of numbers?"

#### Notes

If you show the students the Simone Biles video, you can explain that a mathematician would map Simone's movement by simply tracking the point at her CG. A mathematician reduces a person to a single point! For our CG experiment, we are reducing a balsa-wood stick to a clothespin, and all our clothespins now represent numbers on a weightless stick. Remind the students that, while weightless sticks do not exist in the real world, they are very important tools in the world of mathematics. You can recall the original diagram when we calculated the average of 3, 4, and 11. Those were weightless points, balanced on a weightless stick!

#### Lesson Extension Activities

- Challenge the students to make accurate predictions of the CG by using the smaller 1.75" clothespins. What new information would we need?
- Using four and then five clothespins, challenge the students to create data sets with four or five items in which the CG is a given value. For example, Create and test a data set with four values in which the CG with the clothespins is the same as the CG without the clothespins.



Expand the final discussion at the end of the lesson by asking the students to come up with arguments for both negative and affirmative responses to the question, "Is the balance point an accurate term to use for the average of a set of numbers?" The teacher can then call four or five students up to the front of the class for a Point/Counterpoint Lineup. The first student has one minute to justify an answer of either yes or no. The second student then has one minute to justify the opposite answer in a direct reply to the first student's argument. The third student has a minute to counter argue the second student. Continue down the line.



### Group Task: Balance Point and the Center of Gravity

In this experiment you will be reading, recording, calculating, measuring, and balancing. Make sure at least one student in your group is prepared to do each of these things. For example, the student who is recording needs to have a pencil out and ready, and the student who is calculating needs to have a calculator ready. Everyone in the group will be a THINKER.

# Follow the instructions to answer this question: Is *the balance point* an accurate term to use for the average of a set of numbers?

You will perform the same experiment three times and make notes about what you observe. Finally, each of you will individually will have to explain your answer to the question above.

#### Experiment

1. Use the fulcrum to find the center of gravity of your balsa-wood stick. Use your meter stick to carefully measure the Stick CG as the distance in centimeters from the marked end of the stick to the point on the stick where it balances on the fulcrum. Record the distance to the nearest tenth of a centimeter.

Now for each of your data sets (record your data in the space provided on the second page of this Group Task Sheet.

- 2. Calculate the average of the numbers in your data set.
- 3. Place the balsa-wood stick alongside the meter stick with the marked end of the balsa wood aligned with the zero-centimeter mark on the meter stick.
- 4. For each number in the data set, clip the clothespin on the balsa wood at the corresponding number of centimeters from the marked end.
- 5. Together predict the location of the new CG.
- 6. Carefully find and record the new actual CG.

After completing the above for all three data sets, answer the following questions as a group. Be ready! The teacher will choose one of you to be called on to share your answers with the whole class.

# What conclusion can you make about any differences between your predictions and the actual CG?

How might the original CG of the empty balsa wood stick be related to this problem?



## Group Task Data: Balance Point and the Center of Gravity

The Stick CG is \_\_\_\_\_\_ centimeters from the marked end of the stick.

Data Set #1

10, 20, 60

What is the average? \_\_\_\_\_

What value does your group predict for the CG? \_\_\_\_\_

Where is the actual CG? \_\_\_\_\_

Discuss and then record your groups observations. Was your prediction correct? Why not?

Data Set #2	
3, 10, 30	What is the average?
	What value does your group predict for the CG?
	Where is the actual CG?

Discuss and then record your groups observations. Was your prediction correct? Why not?

Data Set #3	
32, 46, 60	What is the average?
	What value does your group predict for the CG?
	Where is the actual CG?

Discuss and then record your groups observations. Was your prediction correct? Why not?



#### Individual Understanding 1: Balance Point and the Center of Gravity

#### Match the vocabulary words with their definitions.

Vo	cabulary	Descriptions	
1.	balance point	A. The mean value of data values that is calculated by taking th	e sum
		of the data and dividing it by the number of data items.	
2.	center of gravity	<ol><li>A measurement that indicates the tendency of an object to</li></ol>	) twist
		or move in a circle.	
3.	torque	C. A term used to describe the average of a list of numbers.	
		<ol><li>To use arithmetic and algebra to solve a mathematical pro</li></ol>	oblem
4.	average	and get an exact, numerical answer.	
		<ol><li>To use a physical device to find out how large a physical ob</li></ol>	ject is
5.	precision	or how much time passes as an object moves.	
		. To use evidence and observations to formulate a hypo	thesis
6.	prediction	about what may happen when an experiment is repeated.	
_		6. A state of balance when an object is not moving because th	e sum
7.	calculate	of the forces acting on it is zero.	
		I. The location of the average weight of an object; the point at	which
8.	measure	all the weight can be supported.	
_	<b>f</b> ]	A quality that indicates the level of accuracy in a measurem	ent or
9.	fulcrum	calculation. How careful were you?	
10	a au illia viu na	. The world in which we think, the abstract world that is us	sed to
10.	equilibrium	both explore big ideas and model the real world.	. :+
11	math world	In e world in which we live and all the objects and people in	i It.
11.			
12	raalworld		
12.			

One definition is missing. Write the word and your own definition for it here:

#### Answer the following three questions at the end of the lesson.

- 1. <u>Calculate</u> (without actually measuring!) the center of gravity obtained by placing clothespins at the following centimeter marks on your group's stick: 57, 62, and 81. Show your work and circle your answer. Don't forget to use your Stick CG in the calculation!
- 2. Is the balance point in the math world equal to the center of gravity in the real world? Explain why or why not. Answer in complete sentences.
- 3. What is a real-world problem that you know math helps us solve? How is math so helpful if it's different from the real world? Answer in complete sentences.



#### Individual Understanding 2: Balance Point and the Center of Gravity

How to include the weight of the stick in the calculation of center of gravity: Calculation Template

$$\frac{(1 \times \_) + (1 \times \_) + (1 \times \_) + (\_ \times \_)}{(1 + 1 + 1 + \_)}$$

$$\frac{(1 \times \_) + (1 \times \_) + (1 \times \_) + (\_ \times \_)}{(1 + 1 + 1 + \_)}$$

11 East 26<sup>th</sup> Street, New York, NY 10010 momath.org



#### CG Calculation Template, Explanation for the Teacher

How to include the weight of the stick in the calculation of the center of gravity.



explained in the lesson plan, this 1<sup>\*</sup> will change to 1.3.



WEIGHT AND B	ALANCE FOR RIDD	LE: 38		
ITEM	WEIGHT (lbs)	ARM (in)	MOMENT (Ib-in)	
Basic Empty Weight	1686.36	40.38	68089.23	
Nose Baggage (DA 42 NG)				L.
Front Seats	370	37	13690	
Rear Seats	15	73	1095	
Forward Baggage	15	95	1427	
Aft Baggage	-	-	-	
ZERO FUEL WEIGHT	2086.36	40.4	84,309.25	
Main Fuel 40 Gallons	240	48	11520	
RAMP WEIGHT	2326.36	41.2	95,829.23	
Fuel: Start, Taxi, Run-up	8	48	384	
TAKEOFF WEIGHT	2 334.36	41.2	96,213.23	
Fuel Burn 15 Gallons	90	48	4320	
LANDING WEIGHT	2244.36	(40,94)	91,893.23	4
CHANGE IN AIR	CRAFT TO RIDDLE			
New Aircraft-Old Aircraft (△)				
NEW TAKEOFF WEIGHT			1.55	
NEW LANDING WEIGHT				
Altimeter Setting 30.17 Surface Temp 2 °F 31	Surface W _°c Headwind_	ind F	s X-Wind	KT
Pressure Altitude 4845	ft Winds at	2K #	2500 12	KT
Density Altitude 7,800	ft Winds at	IZ K tt	240° 7	KT
P	ERFORMANC	EDATA		
Takeoff Weight 9233	4.36 La	ndina Weia	ht 2244.36 lbs	
Potation SS KIAS O	° Flans An	proach 65	KIAS 30	F
Consider Son Obstacle 7	7 0000 00	12266	1518 3	01
Speed at 50h Obstacle	200		111:17/19	210
Dist 1905 HROLL / 400	ATOTAL AT	rival Runwa	YCIL 1614	_
Departere Rwy ZIL /_	7619 ft Be	est Glide Air	speed 66	KI/
ROC at <u>SOUS</u> It =	t/min VA	98	_ KIAS at _ 2.200	
	MULTI-ENGIN	NE DATA		
Single Engine (S.E.) Rate	of Climb at	ft	-	
Single Engine (ener) man				L.



