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for Innovation in Math Teaching

Creating Color Combos

Visual Modeling of Equivalent Ratios

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Lesson Plan

Grades 5-6

MOMATH
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Table of Contents

Creating Color Combos
Visual Modeling of Equivalent Ratios

Lesson Goals
Proportional relationships are found in many different real world patterns. Proportional relationships that are multiplicative rather than additive are a type of relationship that most students have not previously encountered before middle school. Proportional reasoning is crucial to understanding higher mathematics, including linear equations and beyond. This lesson provides students with the gateway to proportional reasoning—equivalent ratios. Students will use color hues to model, calculate, and communicate equivalent ratios.

This activity can be used as an introductory lesson to a unit on ratios, as a mid-unit, or at the end as a culminating activity.

Student Outcomes
Students create specific color hues by mixing different quantities of two primary colors; they then recreate the same color hues in larger or smaller quantities. By matching color hues of different volumes, they can visualize the meaning of equivalent ratios.

Common Core Practice Standards

CCSS.MATH.CONTENT.6.RP.A. Understand ratio concepts and use ratio reasoning to solve problems.

CCSS.MATH.CONTENT.7.RP.A.2. Recognize and represent proportional relationships between quantities.

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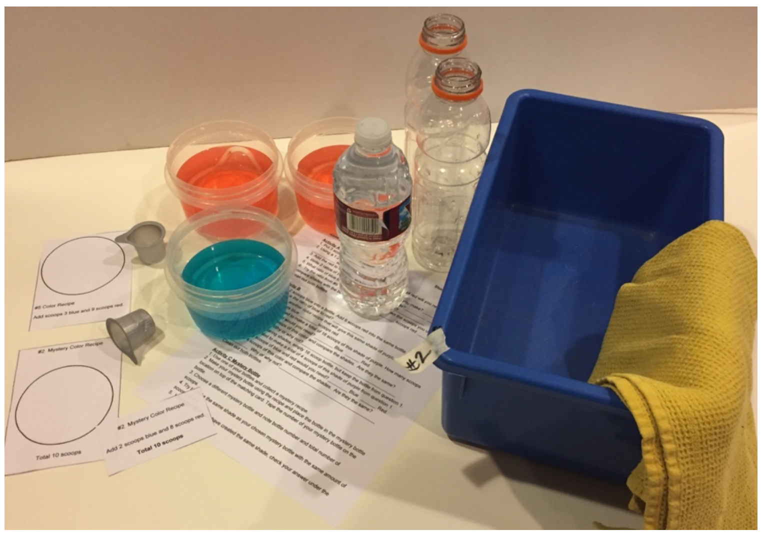
Prerequisite Knowledge

Students should have knowledge of how to multiply two numbers. Some exposure to part-to-part or part-to-total ratios is helpful, but not necessary.

Time Required

Preparation Time: 1.5 hours   Class Time: 50 minutes

Materials



For the whole class, you will need the following materials:

- Food coloring and water (to make 1–2 gallons blue colored water and 3–4 gallons red water).
- 16-20 coffee scoops or empty single-serve creamer containers (these can be purchased or donated by a local gas station).
- 16 empty sports bottles, (16.9 ml with large mouths work best as each needs to hold 16 scoops of water).
- Chart paper, butcher paper, or whiteboard to record equivalent ratios.
- Containers for used water.
- Towels (paper or cloth) for cleanup.
- Question Guide and Student Output sheets.
- Recipe cards for Discovery Activity.
- Mystery recipes and mystery bottle cards.
- Masking tape (8 pieces for mystery bottle).

For each group (table), you will need the following materials:

- Containers of colored water (6 cups of red, 2 cups blue); see mixing instructions.
- Water for rinsing (a large water bottle with a flip-top works well).
- 2 coffee scoops (1- or 2-tablespoon size) or two empty single-serve creamer containers with lip (you will want a few backup creamer containers since they are not as sturdy).
- 2 emptied and rinsed sports drink bottles or other wide mouth bottles.
- Container to dump water (plastic tub or bucket) and small cloth or paper towels for spills.
- 1–4 Student Output sheets (either individual or as a group).
- One recipe card, one mystery recipe card, and one mystery bottle card.

Set Up

In advance, pre-mix two solutions using gallon jugs (either water or rinsed-out juice or milk jugs). For a class of 30, you will likely need one–two gallons of blue water and three–four gallons of red water. Add 16 drops of red coloring to one gallon jug of water and 16 drops of blue coloring to the other. This works out to one drop of food coloring per one cup of water. Each table will need about six cups of red water and two cups of blue water. While it may seem that a darker solution would make the colors easier to identify, the lighter solution allows light in to help distinguish the various hues. It is important to have a uniform mix, both for the mystery solution (see below) and when students compare their initial solution recipes to discover the same hues of equivalent ratios.

Rinse empty sports drink bottles (two per group of four). Water bottles also work, but the sports drink bottles sit sturdier on the tables and have a wider mouth for pouring (reducing accidental spills).

Make an extra copy of mystery recipes. Cut into separate cards. Write the number of the mystery card on the back. Tape so that only the number is showing (back) on a whiteboard, wall, or table. Students will check their recipes for Activity C from the Student Output sheets with these cards

(Front) #1 Mystery Color Recipe Add 4 scoops blue and 2 scoops red. **Total scoops 6**	#1 **(Back)**

Place Mystery Bottle Cards in a central location. This can be the same location as your Discovery Activity bottles, but then it will need to be set up after the Discovery Activity.

Creating Color Combos Part 1

Introduction to the Activity [3 minutes]

Show students one bottle containing red water and another bottle with blue water (filled with obviously different quantities of liquid). Make sure one container will hold both amounts when they are combined. Ask students to share with a partner what they observe. Share a few student responses with the whole class. Ask the students to predict what will happen when the two liquids are combined. Have students combine the bottles of red and blue liquids in front of the class.

Discovery [10 minutes]

Students will discover the relationship between color hues and equivalent ratios.

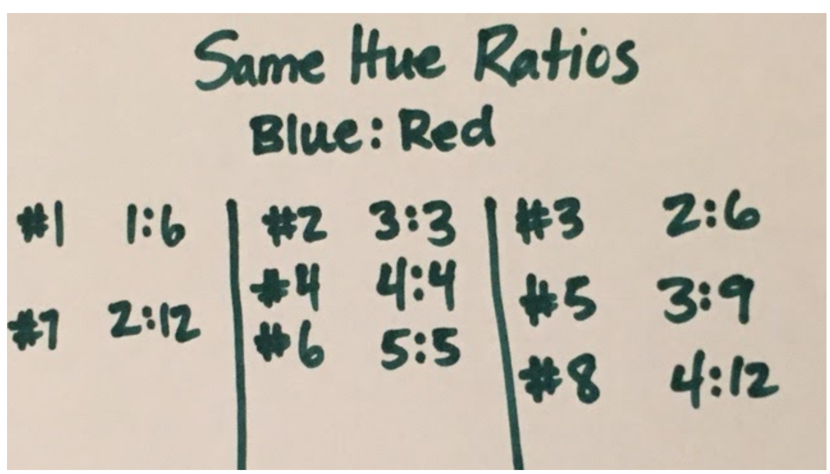
Divide the class into groups of four. Show students how to use the scoops to measure amounts into the bottles, making sure that the scoop is full.

Pass out one color recipe card to each group of four and have them create their recipe.

When they finish, have students place their bottles, mixed according to their specific recipe, on top of their recipe cards in a central location where they are visible for the whole class. A window ledge works well if you have one. Otherwise a common table also works.

Once all bottles containing the mixed recipes are returned, ask students what they observe. When students mention that some are the same hue, sort the bottles by hue (or have a student do so.) Write down on the whiteboard or poster paper (or have another student write down) the color recipes used to create each hue for the class to see. Lead a class discussion of how the color recipe ratios of the same hue are related. Introduce the term “equivalent ratios” as “two ratios that express the same relationship between numbers.”

If you have extra bottles (more than two per group), the students can leave the bottles containing the mixed recipes for reference. Otherwise, have each group retrieve their bottle for the next activity.



Exploration [20–30 minutes]

Students continue to explore the relationship between equivalent ratios.

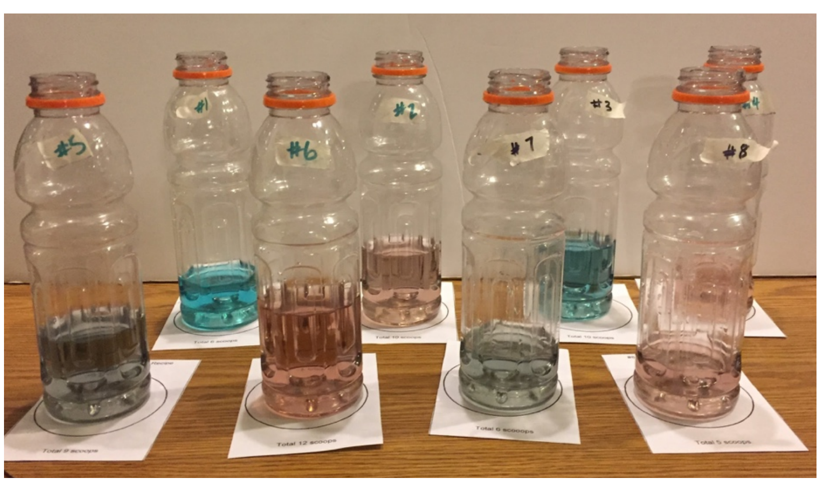
Continue having students work in groups of four. Pass out Student Output sheets. Use one sheet per group (or have each student complete his/her own).

Students will read through the directions and complete the activities. While groups are completing their activities, question student groups to see if they are understanding the relationships. The Questioning Guide can be used to help assess students' current level of understanding and move them forward.

Activity A focuses on equivalent ratio connections to color hue. The 1:2 ratio is easily accessible for most groups. Creating a bluish purple helps students visually connect what happens when the ratio changes.

Activity B requires a more complex understanding of the mathematics of equivalent ratios. Students are asked to make a specific hue and then recreate the same hue with a different volume.

Activity C continues the focus of equivalent ratios of Activity B by creating mystery bottles. Groups first create a mystery bottle, then they try to recreate the mixture of a different group's mystery bottle knowing only the number of scoops used and then comparing the color hue. When groups think they have identified the correct recipe, they check their answer. Groups can then chose another mystery bottle to try and solve. This allows for groups to continue exploring as other groups finish up. There is no need for groups to recreate all mystery bottles.



Summarizing [10 minutes]

After the activity, bring the class together to summarize student discoveries and understanding. Some suggested questions include:

- What kind of relationship is there between equivalent ratios (between each part and between the ratios themselves)? Look at the 1:2 and 3:6 example in Activity A. Ask for other equivalent ratios.
- How can you use the total volume and a color ratio to figure out how much of each color you need to add? (Activity B)

How much information do you need to be able to mix a particular hue of any size batch using two colors? (Activity C)

Exit Ticket (see page 21)

Give Lesson Assessment Exit Ticket: This exit ticket question addresses some of the common misconceptions students have about working with ratios. Students scale ratios both up and down to find equivalent ratios. This also includes the “difference” misconception where students take the difference between 9 and 3 and apply this additive strategy, incorrectly choosing the ratio 2:8. ([b] is the only incorrect response.)

Nya adds 3 scoops blue and 9 scoops red. Which color recipe(s) will give the same hue of purple? Justify your answer(s).

a. 1 scoop blue and 3 scoops red
b. 2 scoops blue and 8 scoops red
c. 6 scoops blue and 18 scoops red
d. 4 scoops blue and 12 scoops red

Essential Questions

- What patterns can help create equivalent ratios?
- How do relationships help us understand and model what we see in the world through mathematics?

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Name_________________________ **Student Output Sheet**

Activity A

1. Put 3 scoops of blue into a bottle.
2. Using a 1:2 ratio for blue to red, how many scoops of red will you need to put in?
____________ scoops red.
3. Add the red to the blue in your bottle, what color did you make? ______________
4. Write two ratios of blue to red that make the same hue? _____:______ ______:______
5. What ratio of blue to red would make a bluish-purple hue? _______:_______
6. Create the ratio from #5 in another bottle and write down the scoops you used and compare hues with the bottle from #3. Scoops blue _________ Scoops red ________
Clean out both bottles.

Activity B

1. Put 2 scoops blue into a bottle. Add 6 scoops red into the same bottle.
2. What is the ratio of blue to red? ________:________
3. What is another color recipe that will give this same hue of purple?
______ scoops blue _________scoops red
4. If you wanted to make a total of 16 scoops of this hue of purple. How many scoops of blue and red would you need? _________ scoops blue _______ scoops red
5. Make 16 scoops of this hue of purple according to your recipe in step 4 and compare the hue to the bottle you created in step 1. Are they the same? ___________ Why or why not?
____
6. After comparing hues, empty the bottle from step 5, but keep the bottle from step 1.
7. If you wanted to make a total of 4 scoops of this hue of purple from step 1, how many scoops of blue and red would you need? _________ scoop blue _______ scoops red.
8. Make 4 scoops of this color according to your recipe in step 7 and compare the hues. Are they the same? ___________ Why or why not? _______________________________
Clean out both bottles.

Activity C: Mystery Bottle

1. Collect a mystery recipe.
2. Mix your mystery bottle per the recipe. Place the bottle in the mystery bottle location on top of the matching card. Tape the number of your mystery bottle on the bottle.
3. Choose a different completed mystery bottle to take back to your group. Note the total number of scoops in the mystery bottle according to the mystery card.
4. In an empty bottle, try to make the same hue as your chosen mystery bottle with the required number of scoops.
5. When you think you have created the same hue, check your answer under the mystery recipe for that number bottle.
6. Return the mystery bottle back to the mystery bottle location on the correct card and choose a different mystery bottle. Repeat the same process.

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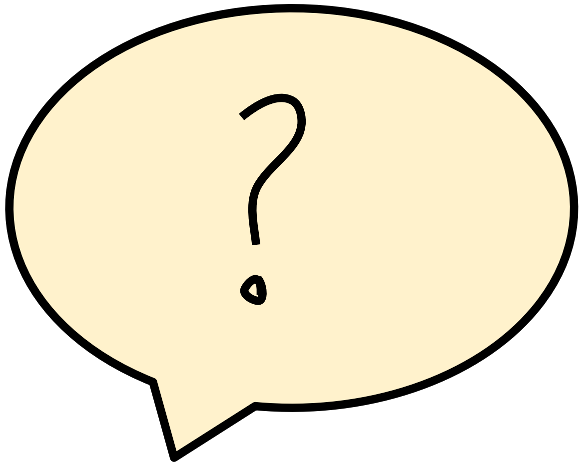
Questioning Guide



This guide will help teachers to assist students with their thinking and to avoid common misconceptions about ratios.

The guide does not include all student responses, but many of the common ones. Making notes on the guide as groups are working about which groups responded in which way will help organize a summary of the activity at the end and allow you to call on students with different strategies to share.

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Questioning Guide for Creating Color Recipes

Activity A

Question: How did you determine how much red you need for 3 scoops of blue?

Response: 1:2; multiply 3 blue scoops by 2 (2 times as much red as blue) to get 3:6

> Question: Is there also a multiplicative relationship between the two ratios themselves? (x3) What does it mean? (Making 3 times the amount.)

Response: 1:2 and 3:6 Multiply both colors by 3

> **Question:** Where else can you find the multiply-by-3 relationship (the number of times 3 scoops goes into 9 scoops). Is there another multiplicative relationship within each ratio? (x2; multiply blue scoops by 2 to get red.)

Response (misconception): 1:2 to 3:4; The student incorrectly used additive approach.

> **Question:** What if you made the 1:2 recipe and your partner also made that same recipe, how many scoops of each would you have? What would the amounts be if you made 3 batches of the color in a 1:2 ratio?

Activity B

Question: How did you find the amount of blue and red in your 16-scoop mixture?

Response: Scale up equivalent ratios until you get to 16. Keep checking by adding the two colors to see if the sum equals 16 scoops.

> Question: Is another way by looking at the final amounts? What do you notice about the total amount of purple? (Have students look at their final mixtures—2 times the original amount of 8 scoops would be 16 scoops.)

Response: Use the scale factor of 2 from total scoops of 8 to 16 to multiply the original mixture (2 blue: 6 red) by 2 to create an equivalent ratio of 4 blue: 12 red.

Response (misconception, only looking at the total amount): Use 8 scoops of blue and 8 scoops of red.

> Question: Is your new mixture more blue or more red (neither.) What about the starting mixture? (more red)
>
> Question: What if you and your partner each made the mixture. Would they be the same hue? Could you combine them? How many scoops of each color would you have?

Activity C

Question: How many total scoops are there? Did you add that many scoops?
Question: Is the color solution you are looking at more red or more blue?
Question: How does this compare to the solution your group made?

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Recipe Cards for Discovery Activity

Students need one card per group of four. These can be laminated if used multiple times. In their groups, students use their unique color recipe to create a color hue and volume in a bottle. Then, students place the bottle on their card in a common area. After all the groups have made their color hues, have a class discussion on how the ratios are related to the color hue with the focus on discovering that equivalent ratios have the same hue. After students finish this activity, they can collect their bottle to be used for the next activity (or if you have extra bottles, 3 per group, they can leave the bottles for reference.)

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#1 Color Recipe Add 1 scoop blue and 6 scoops red.	#2 Color Recipe Add 3 scoops blue and 3 scoops red.
#3 Color Recipe Add 2 scoops blue and 6 scoops red.	#4 Color Recipe Add 4 scoops blue and 4 scoops red.

#5 Color Recipe

Add 3 scoops blue and 9 scoops red.

#6 Color Recipe

Add 5 scoops blue and 5 scoops red.

#7 Color Recipe

Add 2 scoops blue and 12 scoops red.

#8 Color Recipe

Add 4 scoops blue and 12 scoops red.

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Mystery Recipe Cards (Activity C)

Give each group one mystery-bottle recipe card and ask them not to share the recipe with any other group. Make an extra copy of the recipe cards and cut them out. Write the recipe number on the back and tape these to the wall, whiteboard, or desk with only the number side showing. Groups mix their color recipe to create a mystery bottle. Groups then place their mystery bottle in the circle on the matching mystery bottle card in a common area. After students have tried to recreate the mystery hue, they can check their answers by looking under the corresponding mystery bottle recipe number (taped with the number side showing.) These can be laminated if used multiple times.

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#1 Mystery Color Recipe Add 4 scoops blue and 2 scoops red. **Total scoops 6**	#2 Mystery Color Recipe Add 2 scoops blue and 8 scoops red. **Total 10 scoops**
#3 Mystery Color Recipe Add 5 scoops blue and 5 scoops red. **Total 10 scoops**	#4 Mystery Color Recipe Add 1 scoop blue and 5 scoops red. **Total 6 scoops**
#5 Mystery Color Recipe Add 3 scoops blue and 6 scoops red. **Total 9 scoops**	#6 Mystery Color Recipe Add 2 scoops blue and 10 scoops red. **Total 12 scoops**
#7 Mystery Color Recipe Add 2 scoops blue and 4 scoops red. **Total 6 scoops**	#8 Mystery Color Recipe Add 1 scoop blue and 4 scoops red. **Total 5 scoops.**

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Mystery Bottle Cards (Activity C)

Students place their mixed mystery bottle on the matching mystery-bottle card. Groups will then take a different mystery bottle and try to recreate the color hue using the prescribed number of scoops. They will then check their solutions by comparing them both in hue and quantity of liquid with the mystery bottle they chose. Then students will look under cards prepared and taped to the board, wall, or table. After they check their recipe to see if it is a match, they can return the bottle to the mystery bottle location and chose another mystery bottle to try and match.

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#1 Mystery Color Recipe **Total 6 scoops**	#2 Mystery Color Recipe **Total 10 scoops**
#3 Mystery Color Recipe **Total 10 scoops**	#4 Mystery Color Recipe **Total 6 scoops**

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#5 Mystery Color Recipe

Total 9 scoops

#6 Mystery Color Recipe

Total 12 scoops

#7 Mystery Color Recipe

Total 6 scoops

#8 Mystery Color Recipe

Total 5 scoops

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Exit Ticket

Name__________________________ Class_______________ Date______________

Nya adds 3 scoops of blue and 9 scoops of red to a bottle. Which color recipes will give the same hue of purple? Justify your answers.

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Adaptations and Extensions

- Pair up older and younger students for the activity.
- Use test tubes instead of bottles and pipettes or small graduated cylinders instead of scoops to use less water and to practice metric measurement.
- Students create ratio tables for like hues.
- Students graph equivalent ratios with the x- and y-axis, respectively as red scoops and blue scoops.
- Use the activity as a quiz or formative assessment.

For Future Study

Understanding equivalent ratios will allow students to develop an understanding of unit rates, and eventually, the concept of slope of a line. Students will use this same relationship to calculate percent and equivalent fractions. Students model this relationship in future graphs, tables, and equations. Understanding this relationship will help students in later years distinguish between linear but not proportional relationships, inversely proportional relationships, and other non-linear relationships. Equivalent ratios connect to the concept of scale factor in geometry where students use proportions to solve dilations of shapes to create similar figures. Students will also later connect equivalent ratios to trigonometric ratios in right triangles and finding Pythagorean triples.