

MOVES 2015 - Abstracts

Plenary Talks

Elwyn Berlekamp, University of California, Berkeley
The Mathematics of Amazons

John H. Conway, Princeton University
How turning triangles inside out helps us understand their geometry

Richard Guy, University of Calgary
A Triangle has Eight Vertices

Research Talks

Aviv Adler, MIT
The Hardness of Clickomania

Clickomania is a popular computer puzzle game where the player is tasked with clearing a board of colored tiles by removing contiguous same-colored groups. Can it be quickly determined whether a given game is winnable? No; even in very restricted cases, it's NP-complete (and sometimes it's w[1]-hard).

Max Alekseyev, George Washington University
From Silent Circles to Graph Circuits

We present an elegant solution to the question posed by Philip Brocoum, who described the following game as a preliminary event in a drama class he once attended at MIT. This game was played repeatedly by all the students until silence was achieved. An even number of people stand in a circle with their heads lowered. On cue, everyone looks up and stares either at one of their two immediate neighbors (left or right) or at the person diametrically opposed. If two people make eye contact, both will scream! What is the probability that everyone will be silent?

Hossein Behforooz, Utica College
On the Celebration of the 500th Birthday of Melancholia I and the Dürer Magic Square

Magic squares are a major component and the most important part of Recreational Mathematics. Although it is a very old subject, it is still alive and it is growing every day. In this short presentation we will present many interesting properties of a famous four by four Dürer magic square. Then we will use the pattern of this magic square to transform the dates on a calendar and make a magic square. This is a new way to construct magic squares.

Ethan Berkove, Lafayette College
The Colored Cubes Puzzle

A colored cube is one where each face of the cube is one of a proscribed number of colors. Given an arbitrary collection of colored cubes, we investigate when it possible to construct a cube of side length n , each of whose faces is also a single color.

Ben Blum-Smith, Courant Institute of Mathematical Sciences / NYU

Chords of an Ellipse, Fibonacci Numbers, and Cubic Equations

A classic math brain teaser asks for the product of the lengths of certain chords in a unit circle. We consider a variant introduced by Price in which the circle is replaced by an ellipse, leading to a connection with Fibonacci numbers and Lucas polynomials. Our solution is inspired by the classical resolution of the cubic equation by Cardano.

Jeffrey Bosboom, MIT

$1 \times N$ Edge-Matching/Jigsaw Puzzles are NP-Complete

We prove that the centuries-old edge-matching puzzles (square pieces with colored edges which must match) and jigsaw puzzles (square pieces with geometries that must match complementarily) remain NP-complete even if the target shape is just a $1 \times n$ rectangle. This generalizes a previous result of Demaine and Demaine (2007).

Doug Chatham, Morehead State University

The Maximum Queens Problem on a Rectangular Board

If we can first put as many pawns as we like on an m -by- n chessboard, how many queens can we put on that board so that no two queens attack each other? How many pawns are needed to allow that maximum number of queens?

Michael Daly-Jones, Suffolk County Community College

When Game Theory and Sportsmanship Collide: The 2012 Olympic Badminton Scandal

During the 2012 Summer Olympics, four women's doubles teams attempted to gain an unfair advantage by throwing their matches. The design of the tournament, and a suggestion to avoid rewarding unsportsmanlike play, will be examined.

Noam Elkies, Harvard University

On the toroidal crossing number of the complete graph

For every n the complete graph on n vertices can be drawn on the torus with at most $22/243$ as many crossings as there are pairs of edges that might intersect. The constant $22/243 = 0.0905\dots$ improves on $59/648 = 0.0910\dots$ obtained in 1968 by R. K. Guy, T. Jenkyns, and J. Schaer.

Nathan Fox, Rutgers – The State University of New Jersey

Aperiodic Subtraction Games

We will define the Sprague-Grundy function for combinatorial games. Then, we will define subtraction games, a generalization of Nim. Finally, we will construct a subtraction game whose sequence of Sprague-Grundy values, its so-called Nim sequence, is bounded and aperiodic.

Greg Frederickson, Purdue University

Hinged Dissections Inspired by Ernest Freese's Manuscript on Geometric Dissections

Drawing inspiration from hinged dissections in Ernest Freese's 1957 manuscript on geometric dissections, I present attractive new hinged dissections. They include dissections of two similar but different-sized regular polygons to a larger one, and dissections of unusual crosses to squares which extend the use of completing the tessellations techniques.

Nathaniel Friedman, SUNY-Albany

Knots and Soap Film Minimal Surfaces

A knot diagram can be two-colored shaded and white (checker boarding). The checker boarding can predict the soap film minimal surface (sfms) on a corresponding wire model of the knot and whether the sfms is one-sided or two-sided. Examples of sfms's will be presented.

Darren Glass, Gettysburg College

Chutes and Ladders with Large Spinners

In this talk, we look at the effect of the size of a spinner on the length of a game of Chutes and Ladders. In particular, we use a combinatorial approach to prove a conjecture from a 2011 *College Math Journal* article stating that a game on a board with n squares will have the same expected length if you play with a spinner of size n or $n - 1$, regardless of where the chutes and ladders are placed.

Gary Gordon, Lafayette College

Counting sets in SET and flipping coins

We connect fundamental questions about the number of sets sharing k attributes in n -attribute Set to a coin-flipping game. This allows us to answer questions about Set by using Bernoulli trials, the binomial distribution and the central limit theorem. The connection is direct, but somewhat surprising.

Christopher Hanusa, Queens College, CUNY

A q -Queens Problem

We develop a mathematical theory to address the question “In how many ways can you place q chess pieces on a polygonal chessboard so that no two pieces attack each other?”

Brian Hopkins, Saint Peter’s University

Can You Always Win Khalou in Five Moves?

The 2013 smartphone game Khalou presents the player with an array of 16 white and black stones. The goal is to make them all white using various moves which swap the colors of particular stones. Our analysis builds on tools developed for Lights Out and Berlekamp’s Switching Game.

Tim Hsu, San Jose State University

Blue-Red CHOMP

We investigate Blue-Red CHOMP, a partizan variation in which blue (resp. red) nodes in a poset and their upsets can only be removed by Left (resp. Right). Our results include: Every Blue-Red CHOMP game is a number, and 2-level Blue-Red CHOMP is NP-hard.

Craig Kaplan, University of Waterloo

On Surrounding a Polygon

I discuss the geometric problem of surrounding a polygon with copies of itself. This problem arises naturally in tiling theory, in the computation of Heesch Numbers, but can also find a compelling realization as a new style of geometric puzzle.

Tanya Khovanova, MIT

Cookie Monster Plays Games

We research a combinatorial game based on the Cookie Monster problem called the Cookie Monster game that generalizes the games of Nim and Wythoff. We also propose several combinatorial games that are in between the Cookie Monster game and Nim. We discuss properties of P-positions of all of these games.

Justin Kopinsky, MIT

The Parameterized Complexity of Ricochet Robots

We present several results characterizing the parameterized complexity of Ricochet Robots. In particular, we show that Ricochet Robots is $W[1]$ -complete with respect to solution length, and $W[SAT]$ -hard with respect to number of robots.

Yoshiyuki Kotani, Tokyo University of A&T

Variations of the Tower of Hanoi, and Visualization of their State Spaces

I made a few variations of the Tower of Hanoi. For example, a disk cannot visit one of the columns. Their state spaces and the minimal move solutions are shown visually, as well as discussed mathematically.

Urban Larssen, Dalhousie University

Grundy values of Fibonacci Nim

We discuss the classical game of Fibonacci Nim, but now played on several heaps, and with a local move-dynamic rule. Already in 1963 Whinihan published a solution for the one heap game involving the Zeckendorf representation of natural numbers. We extend this result by using the Sprague-Grundy theory.

Kevin Lee, Normandale Community College

Morphing Algorithms and Animations for Escher-like Tessellations

Using TesselManiac!, my new software program for creating Escher like tessellations, I explore linear and non-linear algorithms that automatically morph from base polygon to final tile shape. I will then discuss my cubic spline, path-based editor that gives the artist more control over the morph frames.

Stephen Lucas, James Madison University

Who Wins When Playing Dreidel

Dreidel is a Jewish game with a spinning top taking four values. While past analysis assumed continuous variables, a better approach is a Markov chain on natural numbers. Here, we review relevant aspects of Markov chains, how they can be applied to dreidel, and who is more likely to win.

Jayson Lynch, MIT

The Computational Complexity of Push-Pull Block Puzzles

This paper proves that push-pull block puzzles in 3D are PSpace-Complete to solve and push-pull block puzzles in 2D with thin walls are NP-Hard to solve. The 2D push-pull block puzzle shows up in a number of video games implying further results.

Vince Matsko, University of San Francisco

Periodic Integer Sequences

Second-order recurrence relations generate interesting sequences, such as the renowned Fibonacci sequence. But much rarer are periodic integer sequences generated by second-order recurrences - there are only five essentially distinct classes of periodic integer sequences. In this talk, we investigate periodic integer sequences generated by higher-order recurrence relations.

Alex Meadows, St. Mary's College of Maryland

A New Twist on Wythoff's Game

We propose a new variant on the classic Wythoff's game, using ideas from knot theory to extend the number of piles. The talk will include some ideas of how one analyzes combinatorial games, and some recent results and open problems related to this new game.

David Molnar, Rutgers University

Square-Grid Connection Games

Connection games are a family of two-player abstract strategy games. In the classic *Hex*, the hexagonal grid ensures that one player must win. I will survey various proposals for connection games on a square grid, and examine why one in particular, *Slither*, is so successful.

William Moses, MIT

Computational Complexity of Arranging Music

This paper proves that arrangement of music is NP-hard when subject to various constraints including: avoiding musical dissonance, limiting how many notes can be played simultaneously, and limiting transition speed between chords. These proofs are then used in various applications, including musical choreography and rhythm games such as Rock Band.

Richard Nowakowski, Dalhousie University

Waiting for the Empty Set

If the empty set behaves itself then the corresponding scoring games are also well-behaved.

Eleftherios Pavlides, Roger Williams University

Hinge-elastegrity's shape-shifting into all five platonic solids and the tesseract

Shape shifting hinge-elastegrities (named by analogy to tensegrities= tension-integrity) maintain chiral icosahedral symmetry through elasticity pulsating into octahedra. Created through paper folding and weaving, they form a network of rigid and elastic members. We prove though trig that further folding shape-shifts vertices into cubic, dodecahedral, icosahedral, tetrahedral, and octahedral symmetry.

Charles Petersen, University of California Santa Cruz

Upset-Downset

Upset-downset is a partizan version of CHOMP where Left takes upsets from a finite poset P (i.e., all $y \in P$ such that y is \geq some fixed x) and Right takes downsets. We solve the case where P is a disjoint union of complete bipartite graphs.

James Propp, UMass Lowell

Games of No Strategy and Low-Grade Combinatorics

Some games have the amusing (though sometimes cleverly hidden) feature that the outcome is unaffected by the choices that are made along the way. Such games provide raw material for some fascinating enumerative questions, leading to the frontier of the kinds of counting problems combinatorialists know how to solve.

Alok Puranik, Westford Academy

A Cellular Automaton on a Hexagonal Grid

We examine a cellular automaton on a two-dimensional hexagonal grid. A cell with exactly one live neighbor is born, and live cells never die. The automaton has a fractal structure connected to the Sierpinski sieve. We conjecture a formula for determining which cells are born and when they are born.

David Richeson, Dickinson College

The Mathematics of Tilt

We analyze the game Tilt – ThinkFun's logical puzzle game of sliders and blockers. Is a given configuration winnable? If so, what is the fewest number of tilts required to win? What is this sequence of tilts? How long would it take a young child, tilting the game randomly, to win?

Jason Rosenhouse, James Madison University

The History and Future Of Logic Puzzles

We shall explore the history of logic puzzles, by focusing especially on the contributions of Lewis Carroll and Raymond Smullyan. They showed how these puzzles were not just for recreation, but were powerful teaching tools as well. We shall then consider the possibility of constructing puzzles for non-classical logics.

Paul Salomon and Justin Lanier, John Burroughs School and Georgia Tech

Perfect Polyominoes

The concept of perfect numbers may be extended to polyominoes. In the search for perfect polyominoes, numerous charming and motivating examples can be found, while some sizes may be ruled out on number theoretic grounds. We demonstrate the infinitude of perfectominoes, share perfectominoes of odd size, and introduce open questions.

Carlos Santos, CEAFEL, University of Lisbon

Chess and Combinatorial Game Theory

In some conferences, John Conway mentioned how happy he was when he discovered the value $1/2$ in the context of games. A not artificial chess problem related to the discovery of the dyadics is presented. Its construction was inspired by a mysterious game played by the world champions Capablanca and Lasker.

Alex Schaefer, Binghamton University

Non-Transitive Dice and Directed Graphs

Originally explored by Martin Gardner, Non-Transitive Dice are labeled in a way that produces results which seem to defy probability. I will expand upon this subject and extend it to the setting of directed graphs.

Derek Smith, Lafayette College

Pondering Pirate Puzzles Purposefully

The Pirate Puzzle is perplexing: pirates propose partitioning plundered prizes, with a policy of punishing poorly-preferred proposals, precipitating a pirate's plunge off the plank. We propose a parallel policy, prompting peculiar partition patterns predicated on particular pirate personalities. Professional, Protestant, and Parrot Pirates play prime parts.

Jorge Nuno Silva, University of Lisbon

My Combinatorial Games

I will describe three combinatorial games I created. The first two, JIL and LIM are played with piles of beans. The third, Latin Erdos, looks like a 5×5 Sudoku. I will characterize optimal play for JIL and LIM. Latin Erdos combines combinatorial (impartial) game theory with a theorem from 1935.

Siobhan Roberts, Freelance

Three Sides of the Same Coin: the Intertwined Lives of Berlekamp, Conway, and Guy

Wherein the author triangulates the singular and divergent views of reality belonging to Elwyn Berlekamp, John Horton Conway, and Richard Guy, in an attempt to determine what exactly went down with their legendary decades-long collaboration.

James Tanton, MAA

A Little Thought on Dots and Dashes

A picture speaks a thousand words! Let's see how encoding a sequence of numbers as a simple picture of dots and dashes reveals deep, and now blatantly natural, insights about the sequence and its partner frequency sequence.

Ron Taylor, Berry College

Planar Tanglegrams

In this presentation we discuss the possible arrangements of the popular Tangle toy. We describe the space of planar tangles and consider moves on tangle configurations that define equivalence classes of these tanglegrams.

Bruce Torrence, Randolph-Macon College

Left Center Right - A New Endgame

We explore a simple modification to the rule governing how the dice game Left Center Right ends. The new rule extends the game in a natural way, and leads to some intriguing mathematics.

Robert Vallin, Lamar University

Penney Roulette Games

Penney's Game involves choosing a three-outcome sequence of coin flips. The game is non-transitive (no matter what Player 1's outcome is, Player 2 can put the odds in her favor). Here we look at three versions of Penney's game as applied to (American) Roulette.

Jonathan Weed, MIT

Multinational War is Hard

We consider a multiplayer generalization of the children's card game War and show that the problem of deciding whether one player can win the game is PSPACE-hard. Under standard computational complexity assumptions, therefore, there is no polynomial-time procedure to determine the winner of this multiplayer version of the game.

Peter Winkler, Dartmouth College

Games as Puzzles

Some of the finest and most entertaining puzzles are presented in the form of games, for which you may be asked "Who wins with best play" or "Find a winning strategy for Alice." For one of those that we will present, a little acquaintance with the theory of combinatorial games developed in Berlekamp, Conway & Guy's *Winning Ways* may come in handy.

Wing Hong Tony Wong, Kutztown University of Pennsylvania

Paired Many-to-Many Disjoint Path Covers of Cuboids

In the game Flow Free, one connects matching colored dots with pipes to create a Flow, so that the pipes cover the entire board, and the pipes do not cross or overlap. This motivates the study of paired disjoint path covers on grid graphs and more generally hypercubes and cuboids.

Family Activities

Bruce Bayly, University of Arizona

Hearing Shapes, Seeing Sounds: Exploring Musical Instruments

Musical sounds are mathematically special: their waveforms are determined by the sizes and shapes of the devices that produce them. We will make some unusual instruments using inexpensive materials, and use various means to visualize their vibrations. You may be surprised by some demonstrations, but it's all governed by math!

Hossein Behforooz, Utica College

A Practical Workshop on Magic Squares

In this workshop, we will introduce you a very fun part of recreational mathematics. It is called Magic Squares. Yes it is magic and it is FYE which means "it is for your Mathematical Entertainment and Adventure". After this display and workshop, you will go home with many tables with numbers. Most of these magic squares have many interesting and amazing properties and that is why it is called MAGIC SQUARE. This workshop is open to any grade from elementary school to college students and also parents. Come and join us and have fun. You will love it! Math is FUN.

Brigitte Bentele

A Math Trail in Madison Square Park

Nancy Blachman and Rebecca Mercuri, Julia Robinson Mathematics Festival
A Sample of Mathematical Puzzles

In playing with puzzles perhaps you will discover that mathematics is entertaining.

Skona Brittain, SB Family School

Map Coloring in a Faraway Land

A map-coloring activity based on a story about a king dividing his kingdom among an increasingly larger set of descendants. Participants create map solutions and color complicated already-created maps. More advanced participants work on tori instead of planes. The Four-Color and Seven-Color Theorems are presented.

Tim Chartier and Tanya Chartier and Boluwatife Odumosu, Davidson College

Random Walk through a Comic Strip

This activity will teach random walks in the context of plot development. Participants will roll two dice to determine which of three possible characters, a prince, princess, and dragon, appear in each panel of a comic. Who appears in the story and when? We let math and probability decide.

Christina Chestnut, Stoked on STEAM

It's KNOT fun!

Are we into string theory? Frayed knot. Using games and interactive examples, participants will explore knots and knot theory. The goal is to become familiar with the terminology of knots and to learn new and fun ways of exploring them while developing problem solving and critical thinking skills.

Caren Diefenderfer, Hollins University

Conway's 'Game of Life' for the Twenty First Century

This activity will have three parts. We will begin with observing the variety of possibilities that arise by choosing different starting configurations when playing Conway's 'Game of Life.'

We will then explain the basic rules and explore some paper and pencil examples. We will end by using NetLogo to demonstrate contemporary Life-like cellular automaton games that interest twenty first century social scientists.

Donna Dietz, American University

Spot it![®], and other diversions using projective geometry

A deck of Spot it![®] cards usually has 55 cards, each card having 8 little pictures on it. The magical mathematical fun lies in the fact that if you select any two cards from the deck, exactly one picture is on both of the cards you selected. Come see how it is possible to design such a deck, and enjoy playing various games with them.

Mircea Draghicescu, ITSPHUN LLC

Hands-on Geometry

Learn how to build models of the five "perfect" solids and many other highly symmetric polyhedra that have fascinated humans for thousands of years. Discover interesting facts about these geometric objects and explore their mathematical beauty during this fun, hands-on, mathematical play activity.

Uttam Grandhi, Tisch School of the Arts

Facto-Rail

This is a 2 player game which involves drawing cards at random from a 40 card deck (no face cards) and the player who collects all the cards in 10 factorial (numbers 1 to 10) the earliest wins the game.

Ann Hanson, Columbia College Chicago

Paper Folding for Fun and for Learning

In this workshop, the participants will fold a variety of paper to create a pyramid, a five-pointed star, a hex-a-flex-a-gon and a magic star.

Patrick Honner, Brooklyn Technical High School

Games on Graphs

In this workshop, participants will engage in a variety of one-, two-, and n -person games played on graphs. Through the games we will explore basic and advanced ideas in graph theory, and participants will be encouraged to create their own graph games and variations.

Brian Kronenthal, Kutztown University of Pennsylvania

Disjoint path covers: all about connecting the dots!

Given a (rectangular) chessboard with two marked squares, can you travel from one square to the other, moving vertically and horizontally, by visiting every other square on the board exactly once? When is it impossible? What if there are several pairs of marked squares to connect simultaneously in this way?

Annette Lievaart

Squabbling in the park

Do you recognize this? You walk in the park with your friend or your parent and you see this huge tree. You say to your friend or parent: "That's a big tree, it must be at least 60 feet tall". And your friend or parent says: "No way, it's much taller". Now you have a problem. Who's right? You can keep on squabbling over it, but how can you be sure?

In this family activity we are going to use a land surveying method to approximate the height of things that are not easy to measure with a measuring tape, like trees, statues, buildings. Therefore every participant is going to make his own clinometer using things every Dutch secondary school student has in his pencil case.

Lew Ludwig, Denison College

A Knotty challenge

Take a piece of string, tie a knot in it and glue the ends of the string together into a closed loop, and you have a mathematical knot. This simple process demonstrates the appeal of knot theory - it is very hands-on and accessible to recreational exploration. In this family activity, we will work with a specific representation of a knot - knot mosaics. Using knot kits we will explore the very new area of knot mosaics (2008, Lomonaco and Kauffman) and look at open problems that you can try to solve. This activity is open to everyone; no prior knowledge of knots is required.

Stuart D. Moskowitz, Humboldt State University

Puzzles Make Math Less Puzzling Or How'd That Bunny Vanish?

Use all 8 Common Core Practice Standards to solve mechanical puzzles such as paradoxical geometric vanishes. They are easy to reproduce, but difficult to figure out, yet explainable with basic algebra and geometry. Well use scissors, TI84s, and abstract reasoning to explore and critique, then take a 500 year historical tour of uses and misuses.

Colm Mulcahy, Spelman College

Mathemagic with a Deck of Cards

There seems to be no end to the mathemagical entertainment to be had with a simple deck of cards, drawing from ideas in algebra, discrete mathematics and probability. We'll present a sampling in the spirit of influential writer Martin Gardner. This material can be used to liven up parties and mathematics classes.

Elana Reiser and Michelle Barbaretti, St. Joseph's College

Rock, Paper, Math

Participants will learn basic game theory in relation to the game Rock, Paper, Scissors. The game will be explained and background information will be given. You will participate in a Rock, Paper, Scissors tournament and learn about various known strategies. Together we will try to determine the best strategy.

James Tanton, MAA

Teacup Twists

Don't turn around just once! Just one full turn puts you in a different physical state than two full turns. This tangled twisty fun activity will prove this so - and will have you think twice about twirling an odd number of times on the dance floor!

Violeta Vasilevska, Utah Valley University

Fun with Coloring and Origami

This activity will provide the participants with opportunity to learn about the Two-color map Theorem and fold as well as color some simple origami flat foldable designs.