# MOVES 2015 - Abstracts

## Plenary Talks

**Elwyn Berlekamp, University of California, Berkeley**
***The Game of Amazons***
Abstract coming soon!

**John H. Conway, Princeton University**
***How turning triangles inside out helps us understand their geometry***
Abstract coming soon!

**Richard Guy, University of Calgary**
***A Triangle has Eight Vertices***
Abstract coming soon!

## Research Talks

**Aviv Adler, MIT**
***The Hardness of Clickomania***
Clickomania is a popular computer puzzle game where the player is tasked with clearing a board of colored tiles by removing contiguous same-colored groups. Can it be quickly determined whether a given game is winnable? No; even in very restricted cases, it's NP-complete (and sometimes it's w[1]-hard).

**Max Alekseyev, George Washington University**
***From Silent Circles to Graph Circuits***
We present an elegant solution to the question posed by Philip Brocoum, who described the following game as a preliminary event in a drama class he once attended at MIT. This game was played repeatedly by all the students until silence was achieved. An even number of people stand in a circle with their heads lowered. On cue, everyone looks up and stares either at one of their two immediate neighbors (left or right) or at the person diametrically opposed. If two people make eye contact, both will scream! What is the probability that everyone will be silent?

**Hossein Behforooz, Utica College**
***On the Celebration of the 500th Birthday of Melancholia I and the Dürer Magic Square***
Magic squares are a major component and the most important part of Recreational Mathematics. Although it is a very old subject, it is still alive and it is growing every day. In this short presentation we will present many interesting properties of a famous four by four Dürer magic square. Then we will use the pattern of this magic square to transform the dates on a calendar and make a magic square. This is a new way to construct magic squares.

**Ethan Berkove, Courant Institute of Mathematical Sciences / NYU**
***The Colored Cubes Puzzle***
A colored cube is one where each face of the cube is one of a proscribed number of colors. Given an arbitrary collection of colored cubes, we investigate when it possible to construct a cube of side length $n$, each of whose faces is also a single color.

**Ben Blum-Smith, NYU**

***Chords of an Ellipse, Fibonacci Numbers, and Cubic Equations***

A classic math brain teaser asks for the product of the lengths of certain chords in a unit circle. We consider a variant introduced by Price in which the circle is replaced by an ellipse, leading to a connection with Fibonacci numbers and Lucas polynomials. Our solution is inspired by the classical resolution of the cubic equation by Cardano.

**Jeffrey Bosboom, MIT**

$1 \times N$ ***Edge-Matching/Jigsaw Puzzles are NP-Complete***

We prove that the centuries-old edge-matching puzzles (square pieces with colored edges which must match) and jigsaw puzzles (square pieces with geometries that must match complementarily) remain NP-complete even if the target shape is just a $1 \times n$ rectangle. This generalizes a previous result of Demaine and Demaine (2007).

**Doug Chatham**

***The Maximum Queens Problem on a Rectangular Board***

If we can first put as many pawns as we like on an $m$-by-$n$ chessboard, how many queens can we put on that board so that no two queens attack each other? How many pawns are needed to allow that maximum number of queens?

**Michael Daly-Jones**

***A Fair Method for Executing Tournaments Having a Round-Robin Phase Followed by a Single-Elimination Knockout Phase***

During the 2012 Summer Olympics, four women's doubles teams attempted to gain an unfair advantage by throwing their matches. The design of the tournament, and a suggestion to avoid rewarding unsportsmanlike play, will be examined.

**Noam Elkies, Harvard University**

***On the toroidal crossing number of the complete graph***

For every $n$ the complete graph on $n$ vertices can be drawn on the torus with at most $22/243$ as many crossings as there are pairs of edges that might intersect. The constant $22/243 = 0.0905...$ improves on $59/648 = 0.0910...$ obtained in 1968 by R. K. Guy, T. Jenkyns, and J. Schaer.

**Nathan Fox, Rutgers / The State University of New Jersey**

***Aperiodic Subtraction Games***

We will define the Sprague-Grundy function for combinatorial games. Then, we will define subtraction games, a generalization of Nim. Finally, we will construct a subtraction game whose sequence of Sprague-Grundy values, its so-called Nim sequence, is bounded and aperiodic.

**Greg Frederickson, Purdue University**

***Hinged Dissections Inspired by Ernest Freese's Manuscript on Geometric Dissections***

Drawing inspiration from hinged dissections in Ernest Freese's 1957 manuscript on geometric dissections, I present attractive new hinged dissections. They include dissections of two similar but different-sized regular polygons to a larger one, and dissections of unusual crosses to squares which extend the use of completing the tessellations techniques.

**Nathaniel Friedman, SUNY-Albany**

***Knots and Soap Film Minimal Surfaces***

A knot diagram can be two-colored shaded and white (checker boarding). The checker boarding can predict the soap film minimal surface (sfms) on a corresponding wire model of the knot and whether the sfms is one-sided or two-sided. Examples of sfms's will be presented.

**Darren Glass, Gettysburg College**
***Chutes and Ladders with Large Spinners***

In this talk, we look at the effect of the size of a spinner on the length of a game of Chutes and Ladders. In particular, we use a combinatorial approach to prove a conjecture from a 2011 *College Math Journal* article stating that a game on a board with $n$ squares will have the same expected length if you play with a spinner of size $n$ or $n-1$, regardless of where the chutes and ladders are placed.

**Gary Gordon, Lafayette College**
***Counting sets in SET and flipping coins***

We connect fundamental questions about the number of sets sharing $k$ attributes in $n$-attribute Set to a coin-flipping game. This allows us to answer questions about Set by using Bernoulli trials, the binomial distribution and the central limit theorem. The connection is direct, but somewhat surprising.

**Christopher Hanusa, Queens College, CUNY**
***A $q$-Queens Problem***

We develop a mathematical theory to address the question "In how many ways can you place $q$ chess pieces on a polygonal chessboard so that no two pieces attack each other?"

**Brian Hopkins, Saint Peter's University**
***Can You Always Win Khalou in Five Moves?***

The 2013 smartphone game Khalou presents the player with an array of 16 white and black stones. The goal is to make them all white using various moves which swap the colors of particular stones. Our analysis builds on tools developed for Lights Out and Berlekamp's Switching Game.

**Tim Hsu, San Jose State University**
***Blue-Red CHOMP***

We investigate Blue-Red CHOMP, a partizan variation in which blue (resp. red) nodes in a poset and their upsets can only be removed by Left (resp. Right). Our results include: Every Blue-Red CHOMP game is a number, and 2-level Blue-Red CHOMP is NP-hard.

**Craig Kaplan, University of Waterloo**
***On Surrounding a Polygon***

I discuss the geometric problem of surrounding a polygon with copies of itself. This problem arises naturally in tiling theory, in the computation of Heesch Numbers, but can also find a compelling realization as a new style of geometric puzzle.

**Tanya Khovanova, MIT**
***Cookie Monster Plays Games***

We research a combinatorial game based on the Cookie Monster problem called the Cookie Monster game that generalizes the games of Nim and Wythoff. We also propose several combinatorial games that are in between the Cookie Monster game and Nim. We discuss properties of P-positions of all of these games.

**Justin Kopinsky, MIT**
***The Parameterized Complexity of Ricochet Robots***

We present several results characterizing the parameterized complexity of Ricochet Robots. In particular, we show that Ricochet Robots is W[1]-complete with respect to solution length, and W[SAT]-hard with respect to number of robots.

**Yoshiyuki Kotani, Tokyo University of A&T**
***Variations of the Tower of Hanoi, and Visualization of their State Spaces***

I made a few variations of the Tower of Hanoi. For example, a disk cannot visit one of the columns. Their state spaces and the minimal move solutions are shown visually, as well as discussed mathematically.

**Urban Larssen, Dalhousie University**
***Grundy values of Fibonacci Nim***

We discuss the classical game of Fibonacci Nim, but now played on several heaps, and with a local move-dynamic rule. Already in 1963 Whinihan published a solution for the one heap game involving the Zeckendorf representation of natural numbers. We extend this result by using the Sprague-Grundy theory.

**Kevin Lee, Normandale Community College**
***Morphing Algorithms and Animations for Escher-like Tessellations***

Using TesselManiac!, my new software program for creating Escher like tessellations, I explore linear and non-linear algorithms that automatically morph from base polygon to final tile shape. I will then discuss my cubic spline, path-based editor that gives the artist more control over the morph frames.

**Stephen Lucas, James Madison University**
***Who Wins When Playing Dreidel***

Dreidel is a Jewish game with a spinning top taking four values. While past analysis assumed continuous variables, a better approach is a Markov chain on natural numbers. Here, we review relevant aspects of Markov chains, how they can be applied to dreidel, and who is more likely to win.

**Jayson Lynch, MIT**
***The Computational Complexity of Push-Pull Block Puzzles***

This paper proves that push-pull block puzzles in 3D are PSpace-Complete to solve and push-pull block puzzles in 2D with thin walls are NP-Hard to solve. The 2D push-pull block puzzle shows up in a number of video games implying further results.

**Vince Matsko, University of San Francisco**
***Periodic Integer Sequences***

Second-order recurrence relations generate interesting sequences, such as the renowned Fibonacci sequence. But much rarer are periodic integer sequences generated by second-order recurrences - there are only five essentially distinct classes of periodic integer sequences. In this talk, we investigate periodic integer sequences generated by higher-order recurrence relations.

**Alex Meadows, St. Mary's College of Maryland**
***A New Twist on Wythoff's Game***

We propose a new variant on the classic Wythoffs game, using ideas from knot theory to extend the number of piles. The talk will include some ideas of how one analyzes combinatorial games, and some recent results and open problems related to this new game.

**David Molnar, Rutgers University**
***Square-Grid Connection Games***

Connection games are a family of two-player abstract strategy games. In the classic *Hex*, the hexagonal grid ensures that one player must win. I will survey various proposals for connection games on a square grid, and examine why one in particular, *Slither*, is so successful.

**William Moses, MIT**
***Computational Complexity of Arranging Music***

This paper proves that arrangement of music is NP-hard when subject to various constraints including: avoiding musical dissonance, limiting how many notes can be played simultaneously, and limiting transition speed between chords. These proofs are then used in various applications, including musical choreography and rhythm games such as Rock Band.

**Richard Nowakowski, Dalhousie University**
***Waiting for the Empty Set***

If the empty set behaves itself then the corresponding scoring games are also well-behaved.

**Eleftherios Pavlides, Roger Williams University**
***Hinge-elastegrity's shape-shifting into all five platonic solids vertices and also into hypercube model***

Shape shifting hinge-elastegrities (named by analogy to tensegrities= tension-integrity) maintain chiral icosahedral symmetry through elasticity pulsating into octahedra. Created through paper folding and weaving, they form a network of rigid and elastic members. We prove though trig that further folding shape-shifts vertices into cubic, dodecahedral, icosahedral, tetrahedral, and octahedral symmetry.

**Charles Petersen, University of California Santa Cruz**
***Upset-Downset***

Upset-downset is a partizan version of CHOMP where Left takes upsets from a finite poset $P$ (i.e., all $y \in P$ such that $y$ is $\geq$ some fixed $x$) and Right takes downsets. We solve the case where $P$ is a disjoint union of complete bipartite graphs.

**James Propp, UMass Lowell**
***Games of No Strategy and Low-Grade Combinatorics***

Some games have the amusing (though sometimes cleverly hidden) feature that the outcome is unaffected by the choices that are made along the way. Such games provide raw material for some fascinating enumerative questions, leading to the frontier of the kinds of counting problems combinatorialists know how to solve.

**Alok Puranik, Westford Academy**
***A Cellular Automaton on a Hexagonal Grid***

We examine a cellular automaton on a two-dimensional hexagonal grid. A cell with exactly one live neighbor is born, and live cells never die. The automaton has a fractal structure connected to the Sierpinski sieve. We conjecture a formula for determining which cells are born and when they are born.

**David Richeson, Dickinson College**
***The Mathematics of Tilt***

We analyze the game Tilt – ThinkFun's logical puzzle game of sliders and blockers. Is a given configuration winnable? If so, what is the fewest number of tilts required to win? What is this sequence of tilts? How long would it take a young child, tilting the game randomly, to win?

**Jason Rosenhouse, James Madison University**
***The History and Future Of Logic Puzzles***

We shall explore the history of logic puzzles, by focusing especially on the contributions of Lewis Carroll and Raymond Smullyan. They showed how these puzzles were not just for recreation, but were powerful teaching tools as well. We shall then consider the possibility of constructing puzzles for non-classical logics.

**Paul Salomon John, Burroughs School**
***Perfect Polyominoes***

The concept of perfect numbers may be extended to polyominoes. In the search for perfect polyominoes, numerous charming and motivating examples can be found, while some sizes may be ruled out on number theoretic grounds. We demonstrate the infinitude of perfectominoes, share perfectominoes of odd size, and introduce open questions.

**Carlos Santos, CEAFEL / University of Lisbon**
***Chess and Combinatorial Game Theory***

In some conferences, John Conway mentioned how happy he was when he discovered the value 1/2 in the context of games. A not artificial chess problem related to the discovery of the dyadics is presented. Its construction was inspired by a mysterious game played by the world champions Capablanca and Lasker.

**Alex Schaefer, Binghamton University**
***Non-Transitive Dice and Directed Graphs***

Originally explored by Martin Gardner, Non-Transitive Dice are labeled in a way that produces results which seem to defy probability. I will expand upon this subject and extend it to the setting of directed graphs.

**Derek Smith, Lafayette College**
***Pondering Pirate Puzzles Purposefully***

The Pirate Puzzle is perplexing: pirates propose partitioning plundered prizes, with a policy of punishing poorly-preferred proposals, precipitating a pirate's plunge off the plank. We propose a parallel policy, prompting peculiar partition patterns predicated on particular pirate personalities. Professional, Protestant, and Parrot Pirates play prime parts.

**Jorge Nuno Silva, University of Lisbon**
***My Combinatorial Games***

I will describe three combinatorial games I created. The first two, JIL and LIM are played with piles of beans. The third, Latin Erdos, looks like a $5 \times 5$ Sudoku. I will characterize optimal play for JIL and LIM. Latin Erdos combines combinatorial (impartial) game theory with a theorem from 1935.

**Siobhan Roberts, Freelance**
***Three Sides of the Same Coin***

Wherein the author triangulates the singular and divergent views of reality belonging to Elwyn Berlekamp, John Horton Conway, and Richard Guy, in an attempt to determine what exactly went down with their legendary decades-long collaboration.

**James Tanton, MAA**
***A Little Thought on Dots and Dashes***

A picture speaks a thousand words! Let's see how encoding a sequence of numbers as a simple picture of dots and dashes reveals deep, and now blatantly natural, insights about the sequence and its partner frequency sequence.

**Ron Taylor, Berry College**
***Planar Tanglegrams***

In this presentation we discuss the possible arrangements of the popular Tangle toy. We describe the space of planar tangles and consider moves on tangle configurations that define equivalence classes of these tanglegrams.

**Bruce Torrence, Randolph-Macon College**
***Left Center Right - A New Endgame***

We explore a simple modification to the rule governing how the dice game Left Center Right ends. The new rule extends the game in a natural way, and leads to some intriguing mathematics.

**Robert Vallin, Lamar University**
***Penney Roulette Games***

Penney's Game involves choosing a three-outcome sequence of coin flips. The game is non-transitive (no matter what Player 1's outcome is, Player 2 can put the odds in her favor). Here we look at three versions of Penney's game as applied to (American) Roulette.

**Jonathan Weed, MIT**
***Multinational War is Hard***

We consider a multiplayer generalization of the children's card game War and show that the problem of deciding whether one player can win the game is PSPACE-hard. Under standard computational complexity assumptions, therefore, there is no polynomial-time procedure to determine the winner of this multiplayer version of the game.

**Peter Winkler, Dartmouth College**
***Games as Puzzles***

Some of the finest and most entertaining puzzles are presented in the form of games, for which you may be asked "Who wins with best play" or "Find a winning strategy for Alice." For one of those that we will present, a little acquaintance with the theory of combinatorial games developed in Berlekamp, Conway & Guy's *Winning Ways* may come in handy.

**Wing Hong Tony Wong, Kutztown University of Pennsylvania**
***Paired many-to-many disjoint path covers of cuboids***

In the game Flow Free, one connects matching colored dots with pipes to create a Flow, so that the pipes cover the entire board, and the pipes do not cross or overlap. This motivates the study of paired disjoint path covers on grid graphs and more generally hypercubes and cuboids.

# Family Activities

**Bruce Bayly, University of Arizona**
***Hearing Shapes, Seeing Sounds: Exploring Musical Instruments***

Musical sounds are mathematically special: their waveforms are determined by the sizes and shapes of the devices that produce them. We will make some unusual instruments using inexpensive materials, and use various means to visualize their vibrations. You may be surprised by some demonstrations, but it's all governed by math!

**Hossein Behforooz, Utica College**
***A Practical Workshop on Magic Squares***

In this workshop, we will introduce you a very fun part of recreational mathematics. It is called Magic Squares. Yes it is magic and it is FYE which means "it is for your Mathematical Entertainment and Adventure". After this display and workshop, you will go home with many tables with numbers. Most of these magic squares have many interesting and amazing properties and that is why it is called MAGIC SQUARE. This workshop is open to any grade from elementary school to college students and also parents. Come and join us and have fun. You will love it! Math is FUN.

**Nancy Blachman and Rebecca Mercuri, Julia Robinson Mathematics Festival**
***A Sample of Mathematical Puzzles***

In playing with puzzles perhaps you will discover that mathematics is entertaining.

**Skona Brittain, SB Family School**
***Map Coloring in a Faraway Land***

A map-coloring activity based on a story about a king dividing his kingdom among an increasingly larger set of descendants. Participants create map solutions and color complicated already-created maps. More advanced participants work on tori instead of planes. The Four-Color and Seven-Color Theorems are presented.

**Tim Chartier, Davidson College**
***Random Walk through a Comic Strip***

This activity will teach random walks in the context of plot development. Participants will roll two dice to determine which of three possible characters, a prince, princess, and dragon, appear in each panel of a comic. Who appears in the story and when? We let math and probability decide.

**Christina Chestnut, Stoked on STEAM**
***It's KNOT fun!***

Are we into string theory? Frayed knot. Using games and interactive examples, participants will explore knots and knot theory. The goal is to become familiar with the terminology of knots and to learn new and fun ways of exploring them while developing problem solving and critical thinking skills.

**Caren Diefenderfer, Hollins University**
***Conway's 'Game of Life' for the Twenty First Century***

This activity will have three parts. We will begin with observing the variety of possibilities that arise by choosing different starting configurations when playing Conway's 'Game of Life.'
We will then explain the basic rules and explore some paper and pencil examples. We will end by using NetLogo to demonstrate contemporary Life-like cellular automaton games that interest twenty first century social scientists.

**Donna Dietz, American University**
***Spot it!®, and other diversions using projective geometry***

A deck of Spot it!® cards usually has 55 cards, each card having 8 little pictures on it. The magical mathematical fun lies in the fact that if you select any two cards from the deck, exactly one picture is on both of the cards you selected. Come see how it is possible to design such a deck, and enjoy playing various games with them.

**Mircea Draghicescu, ITSPHUN LLC**
***Hands-on Geometry***

Learn how to build models of the five "perfect" solids and many other highly symmetric polyhedra that have fascinated humans for thousands of years. Discover interesting facts about these geometric objects and explore their mathematical beauty during this fun, hands-on, mathematical play activity.

**Uttam Grandhi, Tisch School of the Arts**
***Facto-Rail***

This is a 2 player game which involves drawing cards at random from a 40 card deck (no face cards) and the player who collects all the cards in 10 factorial (numbers 1 to 10) the earliest wins the game.

**Ann Hanson, Columbia College Chicago**
***Paper Folding for Fun and for Learning***

In this workshop, the participants will fold a variety of paper to create a pyramid, a five-pointed star, a hex-a-flex-a-gon and a magic star.

**Patrick Honner, Brooklyn Technical High School**
***Games on Graphs***

In this workshop, participants will engage in a variety of one-, two-, and $n$-person games played on graphs. Through the games we will explore basic and advanced ideas in graph theory, and participants will be encouraged to create their own graph games and variations.

**Brian Kronenthal, Kutztown University of Pennsylvania**
***Disjoint path covers: all about connecting the dots!***

Given a (rectangular) chessboard with two marked squares, can you travel from one square to the other, moving vertically and horizontally, by visiting every other square on the board exactly once? When is it impossible? What if there are several pairs of marked squares to connect simultaneously in this way?

**Annette Lievaart**
***Squabbling in the park***

Do you recognize this? You walk in the park with your friend or your parent and you see this huge tree. You say to your friend or parent: "Thats a big tree, it must be at least 60 feet tall". And your friend or parent says: "No way, its much taller". Now you have a problem. Who's right? You can keep on squabbling over it, but how can you be sure?
In this family activity we are going to use a land surveying method to approximate the height of things that are not easy to measure with a measuring tape, like trees, statues, buildings. Therefore every participant is going to make his own clinometer using things every Dutch secondary school student has in his pencil case.

**Lew Ludwig, Denison College**
***A Knotty challenge***

Take a piece of string, tie a knot in it and glue the ends of the string together into a closed loop, and you have a mathematical knot. This simple process demonstrates the appeal of knot theory - it is very hands-on and accessible to recreational exploration. In this family activity, we will work with a specific representation of a knot - knot mosaics. Using knot kits we will explore the very new area of knot mosaics (2008, Lomonaco and Kauffman) and look at open problems that you can try to solve. This activity is open to everyone; no prior knowledge of knots is required.

**Stuart Moskowitz, Humboldt State University**
***Puzzles Make Math Less Puzzling Or How'd That Bunny Vanish?***

Use all 8 Common Core Practice Standards to solve mechanical puzzles such as paradoxical geometric vanishes. They are easy to reproduce, but difficult to figure out, yet explainable with basic algebra and geometry. Well use scissors, TI84s, and abstract reasoning to explore and critique, then take a 500 year historical tour of uses and misuses.

**Colm Mulcahy, Spelman College**
***Mathemagic with a Deck of Cards***

There seems to be no end to the mathemagical entertainment to be had with a simple deck of cards, drawing from ideas in algebra, discrete mathematics and probability. We'll present a sampling in the spirit of influential writer Martin Gardner. This material can be used to liven up parties and mathematics classes.

**Elana Reiser, St. Joseph's College**
***Rock, Paper, Math***

Participants will learn basic game theory in relation to the game Rock, Paper, Scissors. The game will be explained and background information will be given. You will participate in a Rock, Paper, Scissors tournament and learn about various known strategies. Together we will try to determine the best strategy.

**James Tanton, MAA**

***Teacup Twists***

Don't turn around just once! Just one full turn puts you in a different physical state than two full turns. This tangled twisty fun activity will prove this so - and will have you think twice about twirling an odd number of times on the dance floor!

**And more to come!**