MO MATH
NATIONAL MUSEUM OF MATHEMATICS

2023 *MoMathlon*

PRACTICE PROBLEMS

Sampled from the 2020 *MoMathlon* Tournament

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PRACTICE TOURNAMENT QUESTION LIST

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2020 *MoMathlon* Question List

Individual Round

1. One day a redwood tree said, "I have been alive during all or part of seven centuries." Rounded to the nearest year, what is the youngest that this tree could be?

2. Count on the fingers of your right hand, starting with 1 at your thumb, 2 at your pointer finger, 3 your middle finger, 4 your ring finger, 5 your pinky, then move back with 6 your ring finger, 7 your middle finger, 8 your pointer, etc. If the fingers are labeled thumb=1, pointer=2, middle =3, ring=4, pinky=5, which finger will 2020 hit?

3. Find the largest integer less than 100 which has exactly 6 positive divisors (including 1 and the number itself).

4. How many zeros are at the end of the product $1 \cdot 2 \cdot 3 \cdots 2019 \cdot 2020$?

5. A boy has as many brothers as he has sisters. His sister has twice as many brothers as she has sisters. How many children are in this family?

6. A 5×5×5 cube is built out of 1×1×1 cubes. If the outside surface of the large cube is painted, how many of the small cubes will have paint on exactly one face?

7. Lola begins a spiral journey by walking one mile east, then two miles north, then three miles west, then four miles south, then five miles east, etc. After she has walked 55 miles, what is the square of the distance from her location to her starting point?

8. If a queen-and-a-quarter quotes a quail-and-a-quarter in a quarter of a day, how many quails can a queen quote in a day?

9. A bull is coming directly towards me at 30 mph. I consider two strategies: I consider running directly away from the bull at a fixed rate of speed, or running directly towards the bull with the same rate of speed. I find that with the first strategy, it will take the bull twice as long to reach me as with the second strategy. What is my running speed in mph?

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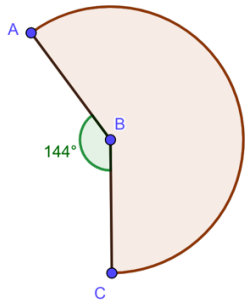
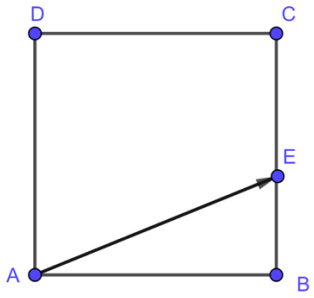
10. A right circular cone is constructed from the circular sector shown below, by carefully aligning AB with CB, so that point B becomes the apex of the cone. If $AB = 55$, what is the height of the cone?



Team Round

11. The average dog sleeps 14 hours each day and lives 12 years. The number of seconds that the average dog spends sleeping during its lifetime has how many digits?

12. A mathematical billiard ball (i.e., a point with zero radius) is shot from corner A of the square of side length 52, and it hits side BC at point E, where $BE = 17$. The billiard then continues to bounce off the walls, until it returns to a corner. How many times will it bounce?



13. How many 4-digit base-10 integers have the property that the digits are increasing from left to right?

14. Find the smallest k such that F_k, the kth Fibonacci number, is a multiple of 1001.

15. The notation $n!$ means $1 \cdot 2 \cdot 3 \cdots n$. For example, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$. Find the remainder when $(1!+2!+3!+\cdots+2020!)^{2020}$ is divided by 6.

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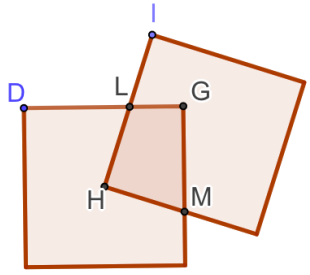
16. Consider two congruent squares, one of whose vertices lies in the center H of the other square. If $DG = 24$ and $\angle ILD = 108^\circ$, find the area of quadrilateral $LGMH$.



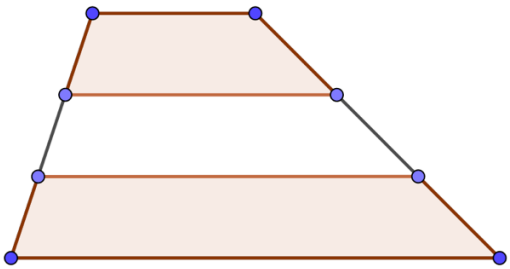
17. Triangle ABC has side lengths AB=5, BC=12, AC=13. A circle with center at D is inscribed in this triangle, and is tangent to side AB at the point E. Find the area of triangle AED.

18. The numbers 1,2,3,4,5 are written on a blackboard on Monday morning. Each night, someone comes up to the blackboard and picks two numbers, and erases them, replacing them with the sum of their sum and their product (for example, the numbers 1 and 5 would be replaced with $1 \cdot 5 + 1 + 5 = 11$.) By Thursday night, there will just be one number left. Let L be the largest possible value for this number, and let S be the smallest possible value. Find $L + S$.

19. Find the largest number that is the product of positive integers whose sum is 22.

Mix-Up Round

20. A trapezoid has a height of 10 with bases of length 23 and 32. Lines are drawn parallel to the bases to divide the trapezoid into 11 strips of equal thickness; the figure below depicts the situation with just 3 strips. If every other strip, starting from the top, is shaded, find the total area of the shaded stripes.



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21. A sock drawer has a huge supply of red, white, and blue socks. You are taking socks out of the drawer in the dark, and cannot see the colors. What is the least number of socks that you must take in order to guarantee that you have taken at least 8 red socks or at least 6 white socks or at least 9 blue socks?

22. Five numbers are in arithmetic progression; in other words, each number in the sequence is equal to the previous number plus a constant. For example, 7, 11, 15, 19 are four numbers in an arithmetic progression. If the first number is 2020 and the third number is 2, what is the sum of the five numbers?

23. Let a/b be the sum, in lowest terms, of

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots\frac{1}{2019\cdot 2020}.$$

Find $a + b$.

24. How many positive integers between 1 and 1001, inclusive, are relatively prime to 1001?

25. Let ABCDE be a 5-digit number. When EDCBA is subtracted from ABCDE, the result is a five-digit number whose first four digits are 3, 4, 9, 4. What is the final digit?

26. Here are some of the entries in a multiplication table that goes up to 12 × 12:

	1	2	⋯	12
1	1	2	⋯	12
2	2	4	⋯	24
⋮	⋮	⋮	⋮	⋮
12	12	24	⋯	144

Find the sum of all the entries. Do not include the labels in the top row or left column.

27. The tetrakaidecahedron is a polyhedron composed of 14 faces: 6 squares and 8 regular hexagons. At each vertex, two hexagons and one square meet. How many vertices does this polyhedron have?

Thank you for your participation! We hope you will come back and enjoy next year's *MoMathlon* tournament. Watch momathlon.momath.org for details.

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PRACTICE TOURNAMENT ANSWER KEY

- **Individual Round**
- **Team Round**
- **Mix-Up Round**

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2020 *MoMathlon* Answer Key

Individual Round

1. 500
2. 4
3. 99
4. 503
5. 7
6. 54
7. 61
8. 4
9. 10
10. 44

Team Round

11. 9
12. 67
13. 126
14. 280
15. 3
16. 144
17. 3
18. 1438

Mix-Up Round

19. 2916
20. 150
21. 21
22. 10
23. 4039
24. 720
25. 7
26. 6084
27. 24

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STUDENT ANSWER SHEETS

CALCULATORS ARE NOT PERMITTED.

Students will be instructed that all answers in these three rounds will be positive integers.

Verbal time warnings will be given when there is approximately half the time remaining, when there are 3 to 5 minutes remaining, and when there is 1 minute remaining. Students should refrain from asking proctors how much time is remaining.

- **Individual Round (10 questions, 35 minutes)**
- **Team Round (9 problems, 25 minutes)**
- **Mix-Up Round (8 problems, 20 minutes)**

In an actual tournament, the timing of the rounds may differ from the timing provided above and may be adjusted at the sole discretion of the Head Proctor to account for several factors. The competition round will finish when the Head Proctor calls an end to the round.

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2020 *MoMathlon* Individual Round

Name _____ Team _____ Team # _____

11. One day a redwood tree said, "I have been alive during all or part of seven centuries." Rounded to the nearest year, what is the youngest that this tree could be?

1)

12. Count on the fingers of your right hand, starting with 1 at your thumb, 2 at your pointer finger, 3 your middle finger, 4 your ring finger, 5 your pinky, then move back with 6 your ring finger, 7 your middle finger, 8 your pointer, etc. If the fingers are labeled thumb=1, pointer=2, middle=3, ring=4, pinky=5, which finger will 2020 hit?

2)

13. Find the largest integer less than 100 which has exactly 6 positive divisors (including 1 and the number itself).

3)

14. How many zeros are at the end of the product $1 \cdot 2 \cdot 3 \cdots 2019 \cdot 2020$?

4)

5. A boy has as many brothers as he has sisters. His sister has twice as many brothers as she has sisters. How many children are in this family?

5)

(Continued on back)

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6. A 5×5×5 cube is built out of 1×1×1 cubes. If the outside surface of the large cube is painted, how many of the small cubes will have paint on exactly one face?

6)

7. Lola begins a spiral journey by walking one mile east, then two miles north, then three miles west, then four miles south, then five miles east, etc. After she has walked 55 miles, what is the square of the distance from her location to her starting point?

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8. If a queen-and-a-quarter quotes a quail-and-a-quarter in a quarter of a day, how many quails can a queen quote in a day?

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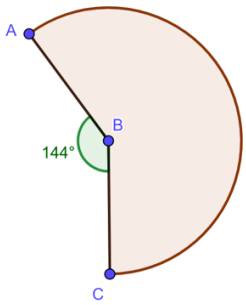

10)

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2020 *MoMathlon* Team Round

Name _____ Team _____ Team # _____

Name _____ Team _____

Name _____ Team _____

Name _____ Team _____

Name _____ Team _____

11. The average dog sleeps 14 hours each day and lives 12 years. The number of seconds that the average dog spends sleeping during its lifetime has how many digits?

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12. A mathematical billiard ball (i.e., a point with zero radius) is shot from corner *A* of the square of side length 52, and it hits side *BC* at point *E*, where *BE* = 17. The billiard then continues to bounce off the walls, until it returns to a corner. How many times will it bounce?

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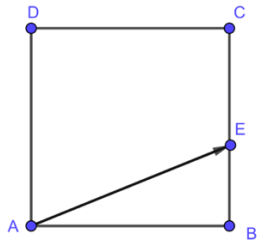

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13)

(Continued on back)

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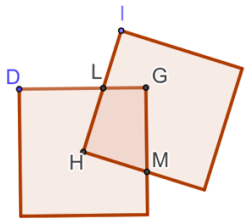

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18)

19. Find the largest number that is the product of positive integers whose sum is 22.

19)

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2020 *MoMathlon* Mix-Up Round

Name _____ Team _____ Team Letter _____

Name _____ Team _____

Name _____ Team _____

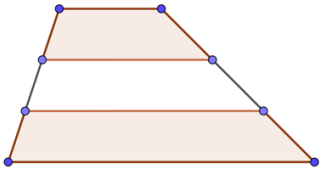
Name _____ Team _____

Name _____ Team _____

Mix-Up Team Spirit Name _____

28. A trapezoid has a height of 10 with bases of length 23 and 32. Lines are drawn parallel to the bases to divide the trapezoid into 11 strips of equal thickness; the figure below depicts the situation with just 3 strips. If every other strip, starting from the top, is shaded, find the total area of the shaded stripes.

20)



29. A sock drawer has a huge supply of red, white, and blue socks. You are taking socks out of the drawer in the dark, and cannot see the colors. What is the least number of socks that you must take in order to guarantee that you have taken at least 8 red socks or at least 6 white socks or at least 9 blue socks?

21)

30. Five numbers are in arithmetic progression; in other words, each number in the sequence is equal to the previous number plus a constant. For example, 7, 11, 15, 19 are four numbers in an arithmetic progression. If the first number is 2020 and the third number is 2, what is the sum of the five numbers?

22)

(Continued on back)

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31. Let a/b be the sum, in lowest terms, of

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots\frac{1}{2019\cdot 2020}.$$

Find $a + b$.

23)

32. How many positive integers between 1 and 1001, inclusive, are relatively prime to 1001?

24)

25. Let ABCDE be a 5-digit number. When EDCBA is subtracted from ABCDE, the result is a five-digit number whose first four digits are 3, 4, 9, 4. What is the final digit?

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26. Here are some of the entries in a multiplication table that goes up to 12 × 12:

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Find the sum of all the entries. Do not include the labels in the top row or left column.

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27. The tetrakaidecahedron is a polyhedron composed of 14 faces: 6 squares and 8 regular hexagons. At each vertex, two hexagons and one square meet. How many vertices does this polyhedron have?

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TIE BREAKER ROUNDS & ANSWER KEY

When two or more students tie for the top scores, those students will be called to the front of the room to answer Tie Breaker questions, one problem per round, with an audience of students and chaperones who will be asked to remain quiet. There may be multiple ties that need to be broken.

In the Tie Breaker rounds, participants will be given a question sheet, which must remain face-down until the instruction is given to start the round. Students will write their answer on the question sheet and draw a rectangular box (Ex. $\boxed{123}$) around their answer. Students will then raise their hand to indicate that they have arrived at an answer and keep their hand raised until the round ends. At the end of each Tie Breaker round, the question sheets will collected by a proctor for scoring.

The last two questions of the Tie Breaker rounds are typically "sudden death" rounds: the first student to submit a correct final answer will win the tie. The round will end when a correct answer has been submitted for all ties that need to be broken.

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2020 *MoMathlon* Tie Breakers

T1. You are given 8 positive integers, the largest of which is 8. The median is 4.5. What is the smallest possible value for the mean of these numbers?

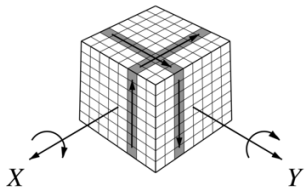
T2. Consider a row of 2,020 seats with N occupants. It turns out that the next person who sits down in this row must sit next to someone. What is the smallest possible value for N?

T3. Twin primes are pairs of prime numbers that are consecutive odd numbers, such as 17 and 19, or 41 and 43. The product of a pair of twin primes equals 55206201D99, where the third-from-last digit is the value D. Find D.

T4. Five numbers are in geometric progression; in other words, each number in the sequence is equal to the previous number multiplied by a constant. For example, 7,21,63,189 are four numbers in a geometric progression. If the first number is 2020 and the third number is 2, what is the product of the five numbers?

T5. Consider a standard clock with an hour-hand, minute-hand, and second-hand. Between 3:30 AM and 3:30 PM, how many times does the second-hand cross the minute-hand?

T6. Consider the 11 × 11 × 11 Rubik's cube below. Each face is painted with a different color, and it is possible to turn any layer, as you can with smaller Rubik's cubes. Let X denote the move that turns the shaded layer shown (indicated by arrows going from the top to the right of the cube) clockwise by 90 degrees, about the axis labeled X. When move X is performed, the only layer that moves is the shaded layer. Likewise, define move Y to be a clockwise 90-degree turn about the axis labeled Y, of just the shaded layer shown (indicated by the arrows going from the front to the top). Let M denote the move "perform X, then perform Y."



Imagine that the cube starts out in "solved" form (so each face has just one color), and we start doing move M repeatedly. What is the least number of repeats of M in order for the cube to be restored to its original colors?

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2020 *MoMathlon* Tie Breakers
Answer Key

Tie Breakers

T1. 3.75
T2. 674
T3. 5
T4. 32
T5. 708
T6. 28

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